FRACTIONAL BERNOULLI POLINOMIALS and APPLICATION

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Abstract

In this paper we discuss Bernoulli numbers and Bernoulli polynomials of order one and order two with shift factor, and their application. Order one Bernoulli polynomials are applied to deconvolution with rectangular apparatus function. Order two Bernoulli polynomials are applied to deconvolution with triangular apparatus function. For trapezoid apparatus function, we define Bernoulli polynomials with parameter δ. Trapezoid apparatus function $\rightarrow$ rectangular apparatus function, if $\delta \rightarrow 0$. Trapezoid apparatus function $\rightarrow$ triangular apparatus function, if $\delta \rightarrow 1$, $(0 \leq \delta \leq 1)$. So,we call this Bernoulli polynomial between order one and order two with parameter δ.

Application

Time-of-flight(TOF) techniques have been useful in studying the velocity distribution of metastably excited and charged particles, the lifetimes of atomic and molecular states, and the kinetics of dissociation processes. Generally, the TOF instrument function is trapezoid.

The relation,

$$F(x) = \int_{-\infty}^{+\infty} f(t')g(t-t')dt'$$

g(z) is the apparatus function of a TOF instrument, F(x) is actual output, f(y) is the original distribution.

For detail, to refer the paper of correction of Instrumental Distortion by Analytical Deconvolution of Data[2].

Conclusion

The convolution integral:

$$h(x) = \int_{-\infty}^{+\infty} f(y)g(x-y)dy = f * g$$

$g(x)$, the apparatus function be given, $h(x)$, the actual instrumental output be measured and $f(x)$, true distribution needs to be found.

$$h(x) = \sum_n a_n x^n,$$

by the MacLauren expansion.

$$f(y) = \sum_n a_n B_n^{(1+\delta)}(t + \frac{1+\delta}{2})$$

$B_n^{(1+\delta)}(t + \frac{1+\delta}{2})$ is relative to the apparatus function $g(x)$.

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