Solutions for Assignment #8

Assignment Information

Maximum grade	20
Due date	November 11, 2004
Instructions	Textbook, pages 129, 130, 145, 146, 164 Problems: 4.1.1 d), e) and f) 4.1.2 b), c) and d) 4.2.1 b) and d) 4.2.5 d) 4.4.2

4.1.1 d)

We assume that L is regular language. $\exists w \in L$, let t be the constant stated in pumping lemma. Consider $w = 0^t1^s2^t \in L$, $|w| = 2t + s >= t$

By the pumping lemma, we can break w = xyz, such that y != ε , |xy| <= t and |y| >= 1.

Since |xy| <=t, y consists of only 0's, we have

$x = 0^{|x|}$

$y = 0^{|y|}$

$z = 0^{t-|xy|}1^s2^t$

Let k = 0

$xy^kz = xy^0z = xz = 0^{|x|}0^{t-|xy|}1^s2^t = 0^{t-|y|}1^s2^t$

Since |y| >= 1, t-|y| < t and t-|y| != t, xz is not in L. Therefore, L is not regular language.

4.1.1 e)

We assume that L is regular language. $\exists w \in L$, let t be the constant stated in pumping lemma. Consider $w=0^t1^s \in L$, $|w| = t + s >= t$

By the pumping lemma, we can break w = xyz, such that y != ε , |xy| <= t and |y| >= 1.

Since |xy| <=t, y consists of only 0's, we have

$x = 0^{|x|}$

$y = 0^{|y|}$

$z = 0^{t-|xy|}1^s$

Let k = s + 1
$xy^kz = xy^{(s+1)}z = xz = 0^{|x|}0^{(s+1)|y|}0^{t-|xy|}1^s = 0^{(t+s)}1^s$

Since t>=1 and t+s>s, $xy^{(s+1)}z$ is not in L. Therefore, L is not regular language.

4.1.1 f)

We assume that L is regular language. ∃w ∈ L, let t be the constant stated in pumping lemma. Consider $w=0^t1^{2t}$∈ L, |w| = 3t >= t

By the pumping lemma, we can break w = xyz, such that y != ε , |xy| <= t and |y| >= 1.

Since |xy| <=t, y consists of only 0's, we have

$x = 0^{|x|}$

$y = 0^{|y|}$

$z = 0^{t-|xy|}1^{2t}$

Let k = 0

$xy^kz = xy^0z = xz = 0^{|x|}0^{t-|xy|}1^{2t} = 0^{t-|y|}1^{2t}$

Since |y|>=1, t-|y| < t and 2(t-|y|) < 2t, xz is not in L. Therefore, L is not regular language.

4.1.2 b)

We assume that L is regular language. Let w=0^n^3 is in L. ∃w ∈ L, let n be the constant stated in pumping lemma. By the pumping lemma, we can break w = xyz, such that y != ε , |xy| <= n and |y| >= 1.

Since |xy| <=n, y consists of between 1 and n 0's.

Let k=2, w=xyyz has length between n^3+1 and n^3+n. Since the next perfect square after n^3 is $(n+1)^3$. Thus, the length of xyyz cannot be a perfect square.

$$(n^3) < (n^3+1) <= |xyyz| <= (n^3+n) < n^3+3n^2+3n+1 = (n+1)^3$$

But if the language were regular, then xyyz would be in the language, which contradicts the assumption that the language of strings of 0's whose length is a perfect cube is a regular language. Therefore, L is not regular language.

4.1.2 c)

We assume that L is regular language. Let w=0^2^k is in L. $\exists$ w $\in$ L, let n be the constant stated in pumping lemma. By the pumping lemma, we can break w = xyz, such that y != ε , |xy| <= n and |y| >= 1. Since |xy| <=n, y consists of between 1 and n 0's.

Let k=$\lceil \lg(n+1) \rceil$ and w=0^2^k, then n = 2^k -1 < 2^k

Consider xyyz whose length is

$$|xyyz| = |xyz| + |y| = 2^k+|y|$$
$$2^k +1 <= |xyyz| <= 2^k +n < 2^k + 2^k$$
$$2^k <= |xyyz| <= 2^{k+1}$$

The length of xyyz is not a power of two, which contradicts the assumption that the language of strings of 0's whose length is a power of 2 is a regular language. Therefore, L is not regular language.

4.1.2 d)

We assume that L is regular language. Let w=$(0+1)^{n^2}$ is in L. $\exists$ w $\in$ L, let n be the constant stated in pumping lemma. By the pumping lemma, we can break w = xyz such that y!= ε , |xy| <= n, and |y|>=1. Since |xy| <=n, |y| is between 1 and n.

Consider k=2,

$$|xy^kz| = |xy^2z| = |xyz| + |y| = |w| + |y| = n^2 + |y|$$

so we have

$$n^2 < n^2 +1 <= |xy^kz| <= n^2 +n < n^2 +2n+1 = (n+1)^2$$

Thus,

The length of $|xy^kz|$ is not a perfect square because n^2 and $(n+1)^2$ are consecutive perfect squares, so there is no perfect square number between them. Therefore, xy^kz is not in this language L. L is not regular language.

4.2.1 b)

→ baababbaa

4.2.1 d)

→ a + abba

4.2.5 d)

From (a), $\frac{d(R+S)}{da} = \frac{dR}{da} + \frac{dS}{da}$

From (b), if $\varepsilon \notin L(R)$, $\frac{d(RS)}{da} = \left(\frac{dR}{da}\right)S$,

if $\varepsilon \in L(R)$, $\frac{d(RS)}{da} = \left(\frac{dR}{da}\right)S + \frac{dS}{da}$

From (c), $\frac{dR*}{da} = \frac{d(\{\varepsilon\})}{da} + \frac{dR}{da} + \frac{d(RR)}{da} + \frac{d(RR^2)}{da} + \frac{d(RR^3)}{da} + \ldots$

$$= \{\} + \frac{dR}{da} + \frac{dR}{da}R + \frac{dR}{da}R^2 + \frac{dR}{da}R^3 + \ldots$$

$$= \frac{dR}{da}(\{\} + R + R^2 + R^3 + \ldots)$$

$$= \frac{dR}{da} \cdot R*$$

Consider T=(0+1)*011=R*S, where R=0+1, S=011. Using rules from (a) to (c), we can obtain,

$$\frac{dT}{da} = \frac{d(R*S)}{da} = \frac{dS}{da} + \frac{d(R*)}{da}S = \frac{dS}{da} + \frac{dR}{da}R*S$$

There are two cases: a=0 and a=1.

If a=0,

$\frac{dS}{da} = \{11\}, \frac{dR}{da} = \{\varepsilon\}$. So, $\frac{dT}{da} = 11 + \varepsilon(0+1)*011 = 11 + (0+1)*011$

If a=1,

$\frac{dS}{da} = \{\}, \frac{dR}{da} = \{\varepsilon\}$. So, $\frac{dT}{da} = \{\} + \varepsilon(0+1)*011 = (0+1)*011$

Therefore, $\frac{d[(0+1)*011]}{da} = \frac{dT}{da} = \begin{cases} 11+(0+1)*011, & \text{if } a=0 \\ (0+1)*011, & \text{if } a=1 \end{cases}$

4.4.2 a)

B	X							
C	X	X						
D		X	X					
E	X		X	X				
F	X	X		X	X			
G		X	X		X	X		
H	X		X	X		X	X	
I	X	X		X	X		X	X
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**

* X indicates pairs of distinguishable states, and the blank squares indicate those pairs that have been found equivalent.

b) the minimum-state equivalent DFA.

	0	**1**
→{A,D,G}	{B,E,H}	{B,E,H}
{B,E,H}	{C,F,I}	{C,F,I}
***{C,F,I}**	{A,D,G}	{B,E,H}