Classification of higher-dimensional operators in the Standard Model

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IMPRS Workshop, Munich 19.07.2010

Effective theories

Standard Model $\rightarrow$ *Extension*

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Appelquist-Carazzone decoupling theorem:

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- Dependencies through EOM

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22 (of 81) operators are redundant and 1 is absent.

SM - gauge group representations structure

Field	representation (dimension)		hypercharge
	SU(3)	SU(2)	U(1)
G_μ	8	1	0
W_μ	1	3	0
B_μ	1	1	0
q	3	2	$\frac{1}{6}$
u	3	1	$\frac{2}{3}$
d	3	1	$-\frac{1}{3}$
l	1	2	$-\frac{1}{2}$
e	1	1	-1
φ	1	2	$\frac{1}{2}$

SM - lagrangian density

$$\begin{aligned}\mathcal{L}_0 = & -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + (D_\mu \varphi)^\dagger (D^\mu \varphi) + m^2 \varphi^\dagger \varphi - \frac{1}{2}\lambda(\varphi^\dagger \varphi)^2 \\ & + i\bar{l}\, \not{D} l + i\bar{e}\, \not{D} e + i\bar{q}\, \not{D} q + i\bar{u}\, \not{D} u + i\bar{d}\, \not{D} d + \\ & - (\bar{l}\Gamma_e e\varphi + \bar{q}\Gamma_u u(\tilde{\varphi}) + \bar{q}\Gamma_d d\varphi + h.c.)\end{aligned}$$

Mass-dimension of fundamental objects in units $\hbar = c = 1$

Type	vector V_μ	tensor $X_{\mu\nu}$	spinor Ψ	skalar φ
Dimension	$(GeV)^1$	$(GeV)^2$	$(GeV)^{\frac{3}{2}}$	$(GeV)^1$
Object	D_μ	$W_{\mu\nu}, G_{\mu\nu}, B_{\mu\nu}$	q, l, u, d, e	φ

- for SU(3)

$$G^A_{\mu\nu} = \partial_\mu G^B_\nu - \partial_\nu G^C_\mu - g_s f^{ABC} G^B_\mu G^C_\nu$$

- for SU(2)

$$W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu - g \varepsilon^{IJK} W^J_\mu W^K_\nu$$

- for U(1)

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Reasoning scheme

1. Description in terms of matter fields φ, ψ, field strength tensors $X_{\mu\nu}$ and covariant derivatives D_μ. Dimensional analysis.
2. Gauge and Lorentz symmetry.
3. Reduction of the set of operators using algebraic properties and SM EOM:

Reasoning scheme

1. Description in terms of matter fields φ, ψ, field strength tensors $X_{\mu\nu}$ and covariant derivatives D_μ. Dimensional analysis.
   e.g. dim-6 expressions containing both fermionic and bosonic fields: $\psi\psi XD$, $\psi\psi X\varphi$, $\psi\psi\varphi\varphi\varphi$, $\psi\psi\varphi\varphi D$, $\psi\psi\varphi DD$, $\psi\psi DDD$
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   many possible choices of ψ - the only singlet in $\hat{2}_{SU(2)} \otimes \hat{2}_{SU(2)}$
   hypercharge conservation

$$(q^\dagger \varepsilon \varphi^*)u, \;\; (q^\dagger \varphi)d, \;\; (l^\dagger \varphi)e, \quad + h.c.$$

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   Lorentz structure contains 2 singlets:

$$(\tfrac{1}{2},0)\otimes(\tfrac{1}{2},0)\otimes(\tfrac{1}{2},\tfrac{1}{2})\otimes(\tfrac{1}{2},\tfrac{1}{2}) = (0,0)\oplus(0,0)\oplus(1,0)\oplus(2,0) \oplus(1,1)\oplus(0,1)\oplus(1,1)\oplus(2,1)$$

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   $$(q^\dagger \varepsilon \varphi^*)u, \quad (q^\dagger \varphi)d, \quad (l^\dagger \varphi)e, \quad + h.c.$$

   2 independent Lorentz invariants (for each):

   $$\bar{\psi}_L \psi_R \varphi D_\mu D^\mu \qquad \bar{\psi}_L \sigma_{\mu\nu} \psi_R \varphi D^\mu D^\nu$$

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3. Reduction of the set of operators using algebraic properties and SM EOM:
   We have (omitting full div) the following operators:

$$(\bar{\psi}_L \sigma_{\mu\nu} \psi_R)(D^\mu D^\nu \varphi) \tag{1}$$

$$(\bar{\psi}_L \sigma_{\mu\nu} D^\mu D^\nu \psi_R)\varphi \tag{2}$$

$$(\bar{\psi}_L \sigma_{\mu\nu} D^\mu \psi_R)(D^\nu \varphi) \tag{3}$$

$$(\bar{\psi}_L D_\mu D^\mu \psi_R)\varphi \tag{4}$$

$$(\bar{\psi}_L D_\mu \psi_R)(D^\mu \varphi) \tag{5}$$

$$(\bar{\psi}_L \psi_R)(D_\mu D^\mu \varphi) \tag{6}$$

Reasoning scheme

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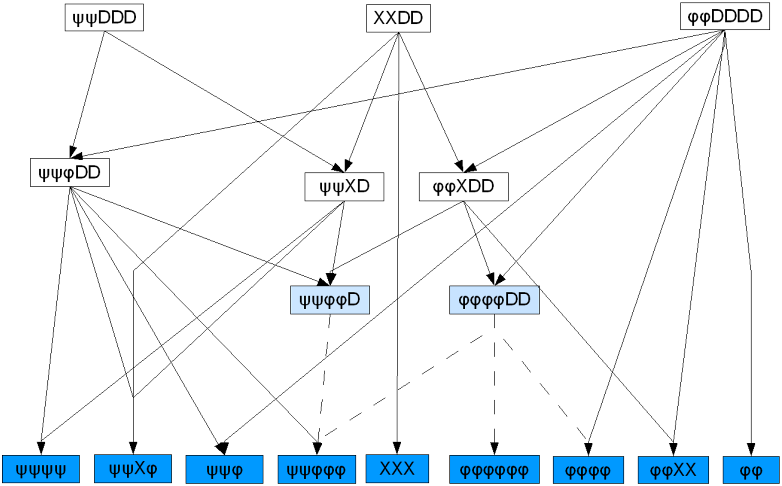
3. Reduction of the set of operators using algebraic properties and SM EOM:
   We can reduce:

$$
\begin{aligned}
(\bar{\psi}_L \sigma_{\mu\nu} \psi_R)(D^\mu D^\nu \varphi) &= \tfrac{1}{2}(\bar{\psi}_L \sigma_{\mu\nu} \psi_R)([D^\mu, D^\nu]\varphi) \\
&= \tfrac{1}{2}(\bar{\psi}_L \sigma_{\mu\nu} \psi_R)(ig\mathcal{W}^{\mu\nu} + ig'\mathcal{B}^{\mu\nu})\varphi \sim \boxed{\psi\psi X \varphi}
\end{aligned}
$$

Reduction scheme



Bosonic invariant operators

φ^6	XXX	$\varphi\varphi XX$	$\varphi^4 DD$
$(\varphi^\dagger\varphi)^3$	$\varepsilon^{IJK} W_\mu^{I\ \nu} W_\nu^{J\ \rho} W_\rho^{K\ \mu}$	$\varphi^\dagger T^I \varphi W^I_{\mu\nu} B^{\mu\nu}$	$(\varphi^\dagger\varphi)(D_\mu\varphi)^\dagger(D^\mu\varphi)$
	$\varepsilon^{IJK} \widetilde{W}^{I\mu\nu} W^J_{\nu\delta} W^{K\delta}_{\ \ \mu}$	$\varphi^\dagger T^I \varphi W^I_{\mu\nu} \tilde{B}^{\mu\nu}$	$[\varphi^\dagger(D^\mu\varphi)][(D_\mu\varphi)^\dagger\varphi]$
	$f^{ABCc} G_\mu^{A\ \nu} G_\nu^{B\ \rho} G_\rho^{C\ \mu}$	$\varphi^\dagger\varphi W^I_{\mu\nu} W^{I\mu\nu}$	
	$f^{ABC} \tilde{G}_\mu^{A\ \nu} G_\nu^{B\ \delta} G_\delta^{C\ \mu}$	$\varphi^\dagger\varphi W^I_{\mu\nu} \tilde{W}^{I\mu\nu}$	
		$\varphi^\dagger\varphi G^A_{\mu\nu} G^{A\mu\nu}$	
		$\varphi^\dagger\varphi G^A_{\mu\nu} \tilde{G}^{A\mu\nu}$	
		$\varphi^\dagger\varphi B_{\mu\nu} B^{\mu\nu}$	
		$\varphi^\dagger\varphi B_{\mu\nu} \tilde{B}^{\mu\nu}$	

Invariant operators with 2 fermions

$\psi\psi\varphi\varphi D$	$\psi\psi\varphi\varphi\varphi$	$\psi\psi X\varphi$
$(\bar{q}\gamma^\mu q)(\varphi^\dagger D_\mu \varphi)$	$[\bar{u}(\tilde{\varphi}^\dagger q)](\varphi^\dagger \varphi)$	$\bar{d}\sigma^{\mu\nu}\lambda^A(\varphi^\dagger q)G^A_{\mu\nu}$
$(\bar{q}\gamma^\mu T^I q)(\varphi^\dagger T^I D_\mu \varphi)$	$[\bar{d}(\varphi^\dagger q)](\varphi^\dagger \varphi)$	$\bar{u}\sigma^{\mu\nu}\lambda^A(\tilde{\varphi}^\dagger q)G^A_{\mu\nu}$
$(\bar{u}\gamma^\mu d)(\varphi^\dagger D_\mu \tilde{\varphi})$	$[\bar{e}(\varphi^\dagger l)](\varphi^\dagger \varphi)$	$\bar{d}\sigma^{\mu\nu}T^I(\varphi^\dagger q)W^I_{\mu\nu}$
$(\bar{u}\gamma^\mu u)(\varphi^\dagger D_\mu \varphi)$		$\bar{u}\sigma^{\mu\nu}T^I(\tilde{\varphi}^\dagger q)W^I_{\mu\nu}$
$(\bar{d}\gamma^\mu d)(\varphi^\dagger D_\mu \varphi)$		$\bar{e}\sigma^{\mu\nu}T^I(\varphi^\dagger l)W^I_{\mu\nu}$
$(\bar{e}\gamma^\mu e)(\varphi^\dagger D_\mu \varphi)$		$\bar{u}\sigma^{\mu\nu}(\tilde{\varphi}^\dagger q)B_{\mu\nu}$
$(\bar{l}\gamma^\mu l)(\varphi^\dagger D_\mu \varphi)$		$\bar{d}\sigma^{\mu\nu}(\varphi^\dagger q)B_{\mu\nu}$
$(\varphi^\dagger l)\gamma^\mu(\bar{l} D_\mu \varphi)$		$\bar{e}\sigma^{\mu\nu}(\varphi^\dagger l)B_{\mu\nu}$

Fermionic operators

$\bar{L}L\bar{L}L$	$\bar{R}R\bar{R}R$
$(\bar{l}_{p_1}\gamma_\mu l_{p_2})(\bar{l}_{p_3}\gamma^\mu l_{p_4})$	$(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e)$
$(\bar{q}_{p_1}\gamma_\mu q_{p_2})(\bar{q}_{p_3}\gamma^\mu q_{p_4})$	$(\bar{u}_{p_1}\gamma_\mu u_{p_2})(\bar{u}_{p_3}\gamma^\mu u_{p_4})$
$(\bar{q}_{p_1}\gamma_\mu T^I q_{p_2})(\bar{q}_{p_3}\gamma^\mu T^I q_{p_4})$	$(\bar{d}_{p_1}\gamma_\mu d_{p_2})(\bar{d}_{p_3}\gamma^\mu d_{p_4})$
$(\bar{q}_{p_1}\gamma_\mu q_{p_2})(\bar{l}_{p_3}\gamma^\mu l_{p_4})$	$(\bar{u}\gamma_\mu u)(\bar{e}\gamma^\mu e)$
$(\bar{q}_{p_1}\gamma_\mu T^I q_{p_2})(\bar{l}_{p_3}\gamma^\mu T^I l_{p_4})$	$(\bar{d}\gamma_\mu d)(\bar{e}\gamma^\mu e)$
	$(\bar{u}_{p_1}\gamma_\mu u_{p_2})(\bar{d}_{p_3}\gamma^\mu d_{p_4})$
	$(\bar{u}_{p_1}\gamma_\mu T^A u_{p_2})(\bar{d}_{p_3} T^A d_{p_4})$

Fermionic operators

$\bar{L}R\bar{R}L$	$\bar{L}R\bar{L}R$
$(\bar{l}e)(\bar{e}l)$	$(\bar{q}_{p_1}u_{p_2})\varepsilon(\bar{q}_{p_3}d_{p_4})$
$(\bar{u}l)(\bar{l}u)$	$(\bar{q}_{p_1}T^A u_{p_2})\varepsilon(\bar{q}_{p_3}T^A d_{p_4})$
$(\bar{d}l)(\bar{l}d)$	$(\bar{q}u)\varepsilon(\bar{l}e)^T$
$(\bar{q}_{p_1}e_{p_2})(\bar{e}_{p_3}q_{p_4})$	$(\bar{q}e)\varepsilon(\bar{l}u)^T$
$(\bar{q}_{p_1}u_{p_2})(\bar{u}_{p_3}q_{p_4})$	
$(\bar{q}_{p_1}T^A u_{p_2})(\bar{u}_{p_3}T^A q_{p_4})$	
$(\bar{q}_{p_1}d_{p_2})(\bar{d}_{p_3}q_{p_4})$	
$(\bar{q}_{p_1}T^A d_{p_4})(\bar{d}_{p_3}T^A q_{p_2})$	
$(\bar{q}d)(\bar{e}l)$	

Redundant operators

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$$(\bar{l}_{p_1}\gamma_\mu T^I l_{p_2})(\bar{l}_{p_3}\gamma^\mu T^I l_{p_4})$$

$$T^I_{ij}T^I_{kl} = \frac{1}{2}\delta_{il}\delta_{kj} - \frac{1}{2N}\delta_{ij}\delta_{kl}$$

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The absent one: $(\bar q e)\varepsilon(\bar l u)^T$

Lepton/baryon nr violating invariant operators

$$(l^T \tilde{\varphi}^*)(\tilde{\varphi}^\dagger l) \quad (1)$$

$$\varepsilon^{abc}(l_{p_1}^T \gamma_\mu u^b)_{p_2} \varepsilon (d_{p_3}^{cT} \gamma^\mu q_{p_4}^a) \quad (2)$$

$$\varepsilon^{abc}(e_{p_1}^T \gamma_\mu q_{p_2}^{ai}) \varepsilon^{ij} (u_{p_3}^{cT} \gamma^\mu q_{p_4}^{bj}) \quad (3)$$

$$\varepsilon^{abc}(e_{p_1}^T u_{p_4}^a)(u_{p_3}^{bT} d_{p_2}^c) \quad (4)$$

$$\varepsilon^{abc}(l_{p_1}^T \varepsilon q_{p_2}^a)(q_{p_3}^{bT} \varepsilon q_{p_4}^c) \quad (5)$$

$$\varepsilon^{abc}(l_{p_1}^T \varepsilon \tau^I q_{p_2}^a)(q_{p_3}^{bT} \varepsilon \tau^I q_{p_4}^c) \quad (6)$$

The importance of classification

How to discover new physics?

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Recent papers with redundant operators:

- J. A. Aguilar-Saavedra, "Single top quark production at LHC with anomalous Wtb couplings", Nucl. Phys. B804 (2008) 160;
- K. Agashe, R. Contino, "Composite Higgs-mediated flavor-changing neutral current", Phys. Rev. D 80, 075016 (2009);
- S. Kanemura, K. Tsumura, "Effects of the anomalous Higgs couplings on the Higgs boson production at the Large Hadron Collider ", Eur. Phys. J. C63 (2009) 11;

Questions

Example: reduction of $(\bar\psi D_\mu\psi)(D^\mu\varphi)$

$$(\bar\psi D_\mu \psi)(D^\mu\varphi) = (\bar\psi D_\nu \eta^{\nu\mu}\psi)(D_\mu\varphi) = (\bar\psi D_\nu \tfrac{1}{2}\{\gamma^\nu,\gamma^\mu\}\psi)(D_\mu\varphi)$$

$$= \tfrac{1}{2}(\bar\psi D_\nu\gamma^\nu\gamma^\mu\psi)(D_\mu\varphi) + \tfrac{1}{2}(\bar\psi\gamma^\mu \underline{\not{D}\psi})(D_\mu\varphi)$$

$$= \boxed{\psi\psi\varphi\varphi D} + \tfrac{1}{2} D_\nu[(\bar\psi\gamma^\nu\gamma^\mu\psi)(D_\mu\varphi)] - \tfrac{1}{2}(\underline{\bar\psi \overleftarrow{\not{D}}}\gamma^\mu\psi)(D_\mu\varphi) + \\ - \tfrac{1}{2}(\bar\psi\gamma^\nu\gamma^\mu\psi)(D_\nu D_\mu\varphi)$$

but

$$(\bar\psi\gamma^\nu\gamma^\mu\psi)(D_\nu D_\mu\varphi) = (\bar\psi(-i)\sigma^{\nu\mu}\psi)(\sum_k X^k_{\nu\mu}\varphi) + (\bar\psi\psi)(\underline{D^\mu D_\mu\varphi})$$

so

$$(\bar\psi_L D_\mu\psi_R)(D^\mu\varphi) = \boxed{\psi\psi\varphi\varphi D} + \boxed{\psi\psi X\varphi} + \boxed{\psi\psi\varphi} + \boxed{\psi\psi\varphi\varphi\varphi} + \boxed{\psi\psi\psi\psi}$$

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$$-\tfrac{1}{2}(\bar\psi\gamma^\nu\gamma^\mu\psi)(D_\nu D_\mu\varphi)$$

but

$$(\bar\psi\gamma^\nu\gamma^\mu\psi)(D_\nu D_\mu\varphi) = (\bar\psi(-i)\sigma^{\nu\mu}\psi)(\sum_k X^k_{\nu\mu}\varphi) + (\bar\psi\psi)(\underline{D^\mu D_\mu\varphi})$$

so

$$(\bar\psi_L D_\mu\psi_R)(D^\mu\varphi) = \boxed{\psi\psi\varphi\varphi D} + \boxed{\psi\psi X\varphi} + \boxed{\psi\psi\varphi} + \boxed{\psi\psi\varphi\varphi\varphi} + \boxed{\psi\psi\psi\psi}$$