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Slew maneuver control for spacecraft equipped with star camera and reaction wheels

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Abstract

A configuration consisting of a star camera, four reaction wheels and magnetorquers for momentum unloading has become standard for many spacecraft missions. This popularity has motivated numerous agencies and private companies to initiate work on the design of an imbedded attitude control system realized on an integrated circuit. This paper provides an easily implementable control algorithm for this type of configuration. The work considers two issues: slew maneuver with a feature of avoiding direct exposure of the camera's CCD chip to the Sun and optimal control torque distribution in a reaction wheel assembly. The attitude controller is synthesized applying the energy shaping technique, where the desired potential function is carefully designed using a physical insight into the nature of the problem. A detailed simulation study shows convincing results for the entire range of operation.

Keywords: Attitude control; Satellite control; Energy control; Quaternion feedback

1. Introduction

A typical configuration of an attitude control system considered for many of low earth orbit spacecraft, consists of a star camera, four reaction wheels and magnetorquers for momentum unloading. The algorithms developed in this paper address two control problems: a slew maneuver and a control torque distribution. It is assumed that full state information is available, i.e. the angular velocity and the attitude can be accessed. The issues related to the attitude determination with a star camera and the momentum dumping are not addressed, however, the interested reader may refer to standard textbooks in the field; Sidi (1997) and Wertz (1990).

The algorithm presented in this paper provides an ability to perform a controlled spacecraft maneuver to the desired attitude without any restrictions on the target attitude and to keep it stabilized in all three axes. The Sun is the most dangerous point in the sky for many payloads, the controller therefore provides a built-in safety mechanism for that. The control torque is distributed among available reaction wheels such that in the Euclidean norm sense the resultant angular momentum of each reaction wheel is kept nearest possible to the nominal value.

The controller proposed in this work uses the energy shaping method. The advantage of this approach is that it provides a physical insight into the design. Stabilization by the energy shaping of a Hamiltonian system was first proposed in mid eighties (van der Schaft, 1986). The control action was the sum of the gradient of potential energy and the dissipative force. Such a control law made the system uniformly asymptotically stable to the desired reference point—the point of minimal potential energy (Nijmeijer and van der Schaft, 1990, Chap. 12). This elegant concept is straightforward in the Euclidean space, nevertheless motion control on an arbitrary differential manifold can only be solved locally in the coordinate neighbourhood. Later, the concept was generalized to a coordinate-free setting on a Riemannian manifold in Koditschek (1989). In this paper the energy shaping method is applied to the attitude control

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Nomenclature

$\boldsymbol{b}$	bore axis of star camera
b_1, b_2, b_3	three components of vector $\boldsymbol{b}$
$\boldsymbol{E}$	unit matrix
$\boldsymbol{e}$	identity quaternion
$\boldsymbol{D}$	distribution matrix in reaction wheels assembly
$\boldsymbol{D}^R$	right pseudo-inverse of $\boldsymbol{D}$
G_f	forbidden geodesics
H	Hamiltonian
$\boldsymbol{h}_w$	angular momentum of reaction wheels
$\bar{\boldsymbol{h}}_w$	nominal value of $\boldsymbol{h}_w$
h_{sat}	saturation value of reaction wheel
$\boldsymbol{J}$	inertia tensor
$\boldsymbol{J}^*$	extended inertia tensor
$\boldsymbol{K}_d$	derivative gain
$\boldsymbol{K}_p$	proportional gain
L	Lagragian
$\boldsymbol{M}_d$	dissipative force
$\boldsymbol{M}_c$	control torque
$\boldsymbol{M}_g$	proportional part of the control torque
$\boldsymbol{M}_p$	generalized moment
M_{g1}, M_{g2}, M_{g3}	components of vector $\boldsymbol{M}_g$
$\boldsymbol{M}_w$	torque generated by the reaction wheel assembly
ω	angular velocity
$\boldsymbol{p}$	conjugate momenta
$\boldsymbol{q}$	attitude quaternion
$\boldsymbol{q}_e$	reference attitude
s	Sun vector
T	kinetic energy
T_s	sampling time
U	potential energy

problem. To broaden the access of this exposition it has been chosen to avoid the use of the notions from the differential geometry. The readers interested in the energy shaping method on differential manifolds are referred to the literature; Koditschek (1989), Wisniewski and Kulczycki (2003).

Fig. 1 depicts the structure of the attitude control proposed in this paper. The input to the system is the reference attitude, the attitude and the rate of the spacecraft. Furthermore, the controller uses information of the Sun vector and the bore axis of the star camera. Four functional blocks compose the controller. Guidance computes the desired potential energy to be artificially generated by the controller, Conservative force delivers the negative gradient of the potential energy, and Damping term calculates a derivative feedback. The fourth block of the controller is the Torque computation, which is a specific implementation for the reaction wheels control. It computes the desired control torque to be generated by the reaction wheels, however, it does not distribute it among the wheels. This task is performed by the Control torque allocation block.

2. Canonical form for a rigid body

To apply the energy shaping, van der Schaft (1986), the rigid body motion is expressed in the canonical form. The standard approach is to use a coordinate neighbourhood, e.g. Euler angles and their conjugate momenta. In this work a global approach is chosen. The unit quaternion $\boldsymbol{q} = [q_1\ q_2\ q_3\ q_4]^{\mathrm{T}}$ and the conjugate momenta $\boldsymbol{p} = [p_1\ p_2\ p_3\ p_4]^{\mathrm{T}}$ parameterize the rotational motion of a rigid body. The idea adopted in this section was addressed earlier in celestial mechanics (Cid and Saturio, 1988; Morton, 1994). The authors studied a canonical transformation $f : \mathbb{R}^{2n} \to \mathbb{R}^{2m}$ with $m > n$. The motion of the rigid body constitutes a special case of this transformation for $m = 4, n = 3$. In other words, the rigid body motion is no longer described locally in a 3D Euclidean space but rather globally in 4Ds. Following this idea the body angular velocity vector gets also an extra dimension, which equals trivially 0 only on the unit sphere $S^3 = \{\boldsymbol{q} \in \mathbb{R}^4 : \boldsymbol{q}^{\mathrm{T}}\boldsymbol{q} = 1\}$.

The kinetic energy of a rigid body rotation depends upon the instant angular velocity $\boldsymbol{\omega}$

$$T = \frac{1}{2}\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{J}\boldsymbol{\omega}, \tag{1}$$

where $\boldsymbol{J}$ denotes the inertia tensor. The angular velocity, more precisely $\boldsymbol{\Omega} := [\boldsymbol{\omega}^{\mathrm{T}}\ 0]^{\mathrm{T}}$ may be regarded as an element of the quaternion division ring. The kinetic energy becomes

$$T = \frac{1}{2}\boldsymbol{\Omega}^{\mathrm{T}}\boldsymbol{J}^*\boldsymbol{\Omega}, \tag{2}$$

where $\boldsymbol{J}^*$ is a block diagonal matrix

$$\boldsymbol{J}^* = \begin{bmatrix} \boldsymbol{J} & \boldsymbol{0} \\ \boldsymbol{0} & J_0 \end{bmatrix} \tag{3}$$

The element J_0 takes in general an arbitrary nonsingular value. Using the standard quaternion parameterizations

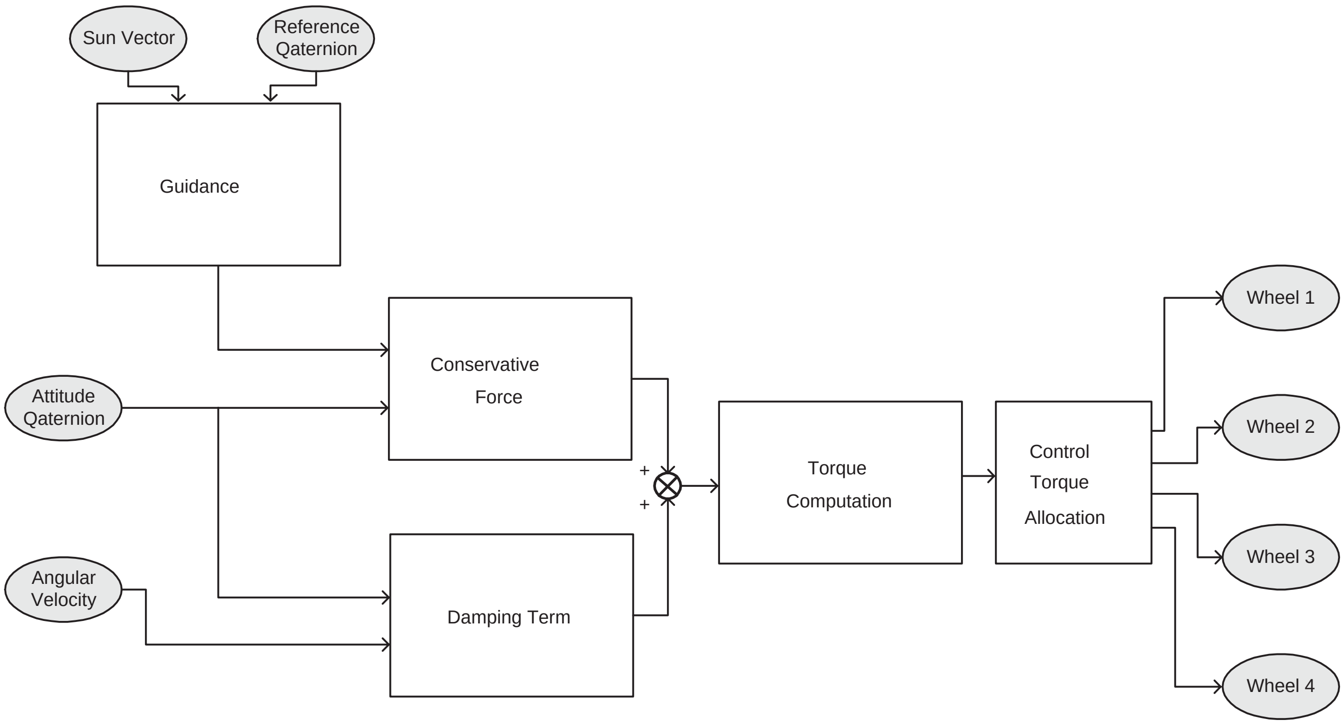

Fig. 1. Scheme of the coarse controller structure.

of kinematics

$$\dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{Q}(\boldsymbol{q})\boldsymbol{\Omega}$$

$$\text{where } \boldsymbol{Q}(\boldsymbol{q}) = \begin{bmatrix} -q_4 & -q_3 & -q_2 & -q_1 \\ -q_3 & -q_4 & -q_1 & -q_2 \\ -q_2 & -q_1 & -q_4 & -q_3 \\ -q_1 & -q_2 & -q_3 & -q_4 \end{bmatrix} \tag{4}$$

the kinetic energy is

$$T = 2\boldsymbol{q}^{\mathrm{T}}\boldsymbol{Q}(\dot{\boldsymbol{q}})\boldsymbol{J}^{*}\boldsymbol{Q}^{\mathrm{T}}(\dot{\boldsymbol{q}})\boldsymbol{q}. \tag{5}$$

Goldstein (1980) gives the following definition of the Hamiltonian:

$$H(\boldsymbol{q},\boldsymbol{p}) = \langle \boldsymbol{p}, \dot{\boldsymbol{q}} \rangle - L(\boldsymbol{q},\dot{\boldsymbol{q}}), \tag{6}$$

where the Lagrangian $L = T(\boldsymbol{q},\dot{\boldsymbol{q}}) - U(\boldsymbol{q})$, and conjugate momentum $\boldsymbol{p}$ is

$$\boldsymbol{p} = \frac{\partial L}{\partial \dot{\boldsymbol{q}}} = \frac{\partial T}{\partial \dot{\boldsymbol{q}}} = 4\dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{J}^{*}\boldsymbol{Q}(\boldsymbol{q}). \tag{7}$$

The Hamiltonian for the rigid body motion becomes then

$$\begin{aligned} H(\boldsymbol{q},\boldsymbol{p}) &= \boldsymbol{p}^{\mathrm{T}}\dot{\boldsymbol{q}} - L(\boldsymbol{q},\boldsymbol{p}) \\ &= \tfrac{1}{8}\boldsymbol{p}^{\mathrm{T}}\boldsymbol{Q}(\boldsymbol{q})\boldsymbol{J}^{*-1}\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{p} + U(\boldsymbol{q}). \end{aligned} \tag{8}$$

Having Hamiltonian the canonical equations are calculated

$$\dot{\boldsymbol{q}} = -\tfrac{1}{4}\boldsymbol{Q}(\boldsymbol{q})\boldsymbol{J}^{*-1}\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{p}, \tag{9}$$

$$\dot{\boldsymbol{p}} = -\tfrac{1}{4}\boldsymbol{Q}(\boldsymbol{p})\boldsymbol{J}^{*-1}\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{p})\boldsymbol{q} - \frac{\partial U(\boldsymbol{q})}{\partial \boldsymbol{q}} + \boldsymbol{M}_p,$$

where $\boldsymbol{M}_p$ stands for the generalized moment.

The spacecraft control torque is denoted by $\boldsymbol{M}_c$. To find the correspondence between the generalized moment and the control torque, the invariance of the work can be used. It follows that the time derivatives of the work done by the torque $\boldsymbol{M}_p$ and $\boldsymbol{M}_c$ are equal

$$\dot{\boldsymbol{q}}^{\mathrm{T}}(t)\boldsymbol{M}_p(t) = \dot{W}(t) = \boldsymbol{\Omega}^{\mathrm{T}}(t)\boldsymbol{M}_c(t). \tag{10}$$

Applying Eq. (4), the right hand side of Eq. (10) is parameterized by $\dot{\boldsymbol{q}}$

$$\dot{\boldsymbol{q}}^{\mathrm{T}}(t)\boldsymbol{M}_p(t) = \dot{W}(t) = 2\dot{\boldsymbol{q}}^{\mathrm{T}}(t)\boldsymbol{Q}(\boldsymbol{q}(t))\boldsymbol{M}(t)$$
$$\text{where } \boldsymbol{M} = [\boldsymbol{M}_c^{\mathrm{T}} \quad 0]^{\mathrm{T}} \tag{11}$$

hence

$$\boldsymbol{M}_p(t) = 2\boldsymbol{Q}(\boldsymbol{q}(t))\boldsymbol{M}(t) \tag{12}$$

or equivalently

$$\boldsymbol{M}(t) = \tfrac{1}{2}\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{q}(t))\boldsymbol{M}_p(t). \tag{13}$$

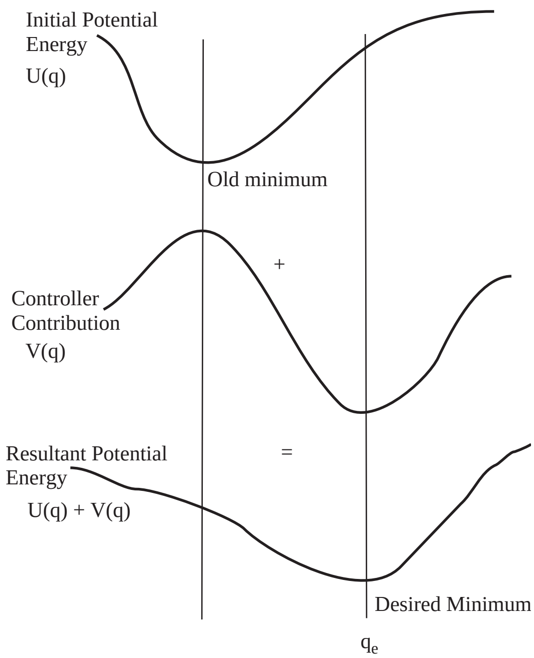

Fig. 2. A control torque equal the negative gradient of potential energy gives a contribution to the total potential energy in the system.

3. Attitude control

The energy shaping in van der Schaft (1986) puts forward a feedback control of the form

$$\boldsymbol{M}_p = -\frac{\partial V(\boldsymbol{q})}{\partial \boldsymbol{q}} + \boldsymbol{M}_d, \tag{14}$$

where $V : S^3 \to \mathbb{R}$ is a continuously differentiable function. The term $\boldsymbol{M}_d$ denotes a dissipative force. The time derivative of its work $\dot{W} = \boldsymbol{M}_d^{\mathrm{T}} \dot{\boldsymbol{q}}$ is negative definite. Assuming as in Fig. 2 the minimum of the potential energy $U(\boldsymbol{q}) + V(\boldsymbol{q})$ at the point $\boldsymbol{q}_e$, the control law (14) makes the system asymptotically stable to the equilibrium point $(\boldsymbol{q}_e, \boldsymbol{0})$.

3.1. Control synthesis

The controller proposed in this paper applies for a spacecraft equipped with a star camera, which bore axis shall never point to the Sun. This attitude is treated as forbidden. For simplicity of the exposition it is assumed that the reference quaternion $\boldsymbol{q}_e = \boldsymbol{e}$, where $\boldsymbol{e} = [0\,0\,0\,1]^{\mathrm{T}}$. Otherwise the quaternion $\boldsymbol{q}$ shall be substituted by $\boldsymbol{Q}(\boldsymbol{q}_e)\boldsymbol{q}$ in the subsequent formulas.

Forbidden attitudes in the slew maneuver problem are not only a certain point $\boldsymbol{q}_f$ on the 3-sphere, but rather the whole geodesics of forbidden attitudes: Having a forbidden attitude $\boldsymbol{q}_f$ the whole family can be generated by a product with a rotation about the bore axis. The control law proposed shall make use of two orthogonal vectors $\boldsymbol{W}_1, \boldsymbol{W}_2 \in \mathbb{R}^4$ normal to the plane defined by the geodesics G_f. They are constructed in the following procedure:

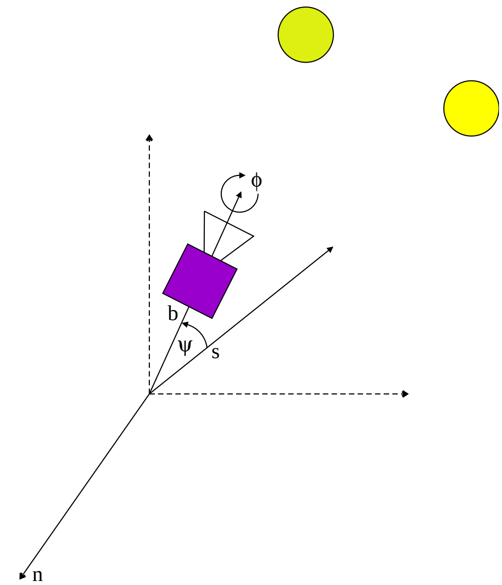

Fig. 3. The variable q_f represents attitude quaternion defining the rotation of the sun vector $\boldsymbol{s}$ to the bore axis $\boldsymbol{b}$.

Procedure 1. *Having determined the unit vector* $\boldsymbol{b}$ *in the direction of the bore axis and the unit vector* $\boldsymbol{s}$ *pointing towards the Sun, a forbidden quaternion can be taken corresponding to the rotation* $\boldsymbol{R}_f : \mathbb{R}^3 \to \mathbb{R}^3$, $\boldsymbol{b} = \boldsymbol{R}_f(\boldsymbol{s})$, *see* Fig. 3. *For this purpose a definition of a unit quaternion in* Goldstein (1980) *is employed*

$$\boldsymbol{q}_f = \left[n_1 \sin\frac{\psi}{2} \quad n_1 \sin\frac{\psi}{2} \quad n_3 \sin\frac{\psi}{2} \quad \cos\frac{\psi}{2} \right]^{\mathrm{T}}, \tag{15}$$

where the triad $\boldsymbol{n} = [n_1\ n_2\ n_3]^{\mathrm{T}}$ *is the unit vector of the rotation axis and* ψ *is the angle of rotation. The vector* $\boldsymbol{n}$ *is orthogonal to* $\boldsymbol{s}$ *and* $\boldsymbol{b}$, $\boldsymbol{n} = \boldsymbol{b} \times \boldsymbol{s} / |\boldsymbol{b} \times \boldsymbol{s}|$. *The angle* $\psi \in [0, \pi]$ *is computed using the scalar product of* $\boldsymbol{s}$ *and* $\boldsymbol{b}$, $\psi = \mathrm{acos}(\boldsymbol{s} \cdot \boldsymbol{b})$.

The geodesics G_f *is the product of* $\boldsymbol{q}_f$ *and the quaternions corresponding to the rotations about the bore axis* $\boldsymbol{b}$

$$G_f = \left\{ \boldsymbol{Q}(\boldsymbol{q}_f) \begin{bmatrix} b_1 \sin\phi \\ b_2 \sin\phi \\ b_3 \sin\phi \\ \cos\phi \end{bmatrix} : \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \boldsymbol{b}, \phi \quad \in [-\pi, \pi) \right\}. \tag{16}$$

Let $\boldsymbol{W}_3 = [b_1\ b_2\ b_3\ 0]^{\mathrm{T}}$ *and* $\boldsymbol{W}_4 = \boldsymbol{e}$. *The vectors* $\boldsymbol{W}_3$ *and* $\boldsymbol{W}_4$ *are orthogonal and belong to the geodesics* G_f. *The desired vectors* $\boldsymbol{W}_1$ *and* $\boldsymbol{W}_2$ *are chosen to form together* *with $\boldsymbol{W}_3$ and $\boldsymbol{W}_4$ orthonormal bases for $\mathbb{R}^4$*

$$\boldsymbol{W}_1 = \boldsymbol{Q}(\boldsymbol{q}_f)[c_1\ c_2\ c_3\ 0]^{\mathrm{T}}, \quad \boldsymbol{W}_2 = \boldsymbol{Q}(\boldsymbol{q}_f)[d_1\ d_2\ d_3\ 0]^{\mathrm{T}}, \tag{17}$$

where $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{d}$ constitute orthonormal bases in $\mathbb{R}^3$. By the construction, the vectors $\boldsymbol{W}_1$, $\boldsymbol{W}_2$ are orthogonal and normal to the plane spanned by the geodesics G_f.

The procedure above provides a definition of two orthonormal vectors $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$, which have a remarkable feature that

$$\boldsymbol{q} \in G_f \iff (\boldsymbol{q}^{\mathrm{T}}\boldsymbol{W}_1)^2 + (\boldsymbol{q}^{\mathrm{T}}\boldsymbol{W}_2)^2 = 0. \tag{18}$$

A potential function $V(\boldsymbol{q})$ suggested for the feedback is

$$V(\boldsymbol{q}) = \frac{k_p(1-q_4)}{(\boldsymbol{q}^{\mathrm{T}}\boldsymbol{W}_1)^2 + (\boldsymbol{q}^{\mathrm{T}}\boldsymbol{W}_2)^2}, \tag{19}$$

where k_p stands for a positive real serving as a design parameter. The function V has the minimum for $\boldsymbol{q} = \boldsymbol{e}$ and diverges to infinity for $\boldsymbol{q} \in G_f$. It is expected that the control torque conforming to Eqs. (13) and (14) will be repellent to the geodesics G_f and the system becomes globally asymptotically stable to the identity $\boldsymbol{e}$. The explicit form for the proportional part of the control torque is

$$\begin{bmatrix} M_{g1} \\ M_{g2} \\ M_{g3} \\ M_{g4} \end{bmatrix} = -\frac{1}{2}\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{q})\frac{\partial V(\boldsymbol{q})}{\partial \boldsymbol{q}} = -\frac{\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{q})((\tilde{q}_1^2+\tilde{q}_2^2)\boldsymbol{e} + 2(1-q_4)(\tilde{q}_1\boldsymbol{W}_1 + \tilde{q}_2\boldsymbol{W}_2))}{2(\tilde{q}_1^2+\tilde{q}_2^2)^2}, \tag{20}$$

where $\tilde{q}_j = \boldsymbol{q}^{\mathrm{T}}\boldsymbol{W}_j$.

The control torque turns out to be

$$\boldsymbol{M}_c = \begin{bmatrix} M_{g1} \\ M_{g2} \\ M_{g3} \end{bmatrix} + \boldsymbol{K}_d\boldsymbol{\omega}. \tag{21}$$

The gain $\boldsymbol{K}_d$ is a negative definite matrix. Notice that the time derivative of the work done by the field $\boldsymbol{K}_d\boldsymbol{\omega}$

$$\dot{W} = \boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{K}_d\boldsymbol{\omega} \tag{22}$$

happens to be negative definite.

This subsection is wrapped up by reformulating Eqs. (19) and (20) for an arbitrary reference $\boldsymbol{q}_e$. The nominator of the potential energy is modified

$$V(\boldsymbol{q}) = k_p\frac{1-\boldsymbol{e}^{\mathrm{T}}\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{q}_e)\boldsymbol{q}}{(\boldsymbol{q}^{\mathrm{T}}\boldsymbol{W}_1)^2 + (\boldsymbol{q}^{\mathrm{T}}\boldsymbol{W}_2)^2}. \tag{23}$$

Then Eq. (21) gives the control law, but now

$$\begin{bmatrix} M_{g1} \\ M_{g2} \\ M_{g3} \\ M_{g4} \end{bmatrix} = -\frac{k_p\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{q})((\tilde{q}_1^2+\tilde{q}_2^2)\boldsymbol{q}_e + 2(1-\boldsymbol{q}_e^{\mathrm{T}}\boldsymbol{q})(\tilde{q}_1\boldsymbol{W}_1 + \tilde{q}_2\boldsymbol{W}_2))}{2(\tilde{q}_1^2+\tilde{q}_2^2)^2}. \tag{24}$$

3.2. Control torque command

To implement the control law in Eqs. (21) and (20) for a spacecraft actuated by reaction wheels, an additional computation has to be carried out. The term $\boldsymbol{\omega} \times \boldsymbol{h}_w$, where $\boldsymbol{h}_w$ is the angular momentum vector contributing from all 4 reaction wheels, has to be feed-forwarded by the controller. This ought to be done in order to incorporate the angular momentum of the wheels in dynamics of the spacecraft. As a result, the torque generated by the wheels becomes

$$\boldsymbol{M}_w = \boldsymbol{M}_c + \boldsymbol{\omega} \times \boldsymbol{h}_w. \tag{25}$$

3.3. Control algorithm

(1) Compute the damping term

$$\boldsymbol{M}_{Damping} = \boldsymbol{K}_d\boldsymbol{\omega}. \tag{26}$$

(2) Compute the conservative term

$$\boldsymbol{M}_{Conservative} = -\boldsymbol{K}_p[M_{g1}\ M_{g2}\ M_{g3}]^{\mathrm{T}}, \tag{27}$$

where

$$\begin{bmatrix} M_{g1} \\ M_{g2} \\ M_{g3} \\ M_{g4} \end{bmatrix} = \frac{\boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{q})((\tilde{q}_1^2+\tilde{q}_2^2)\boldsymbol{q}_e + 2(1-\boldsymbol{q}_e^{\mathrm{T}}\boldsymbol{q})(\tilde{q}_1\boldsymbol{W}_1 + \tilde{q}_2\boldsymbol{W}_2))}{-2(\tilde{q}_1^2+\tilde{q}_2^2)^2}. \tag{28}$$

(3) Compute the angular momentum compensation

$$\boldsymbol{M}_{Compensation} = \boldsymbol{\omega} \times \boldsymbol{h}_w. \tag{29}$$

(4) Compute the control torque

$$\boldsymbol{M}_{Control} = \boldsymbol{M}_{Damping} + \boldsymbol{M}_{Conservative} + \boldsymbol{M}_{Compensation}. \tag{30}$$

Table 1
Spacecraft parameters and control settings used in simulation study

Principal moments of inertia	$I_{xx} = 5,\ I_{yy} = 6,\ I_{zz} = 7\ \text{kgm}^2$
The maximum angular momentum of the reaction wheel	0.12 Nms
The maximum wheel speed	280 rad/s
Moments of inertia of the reaction wheel	0.00043 kgm^2
The nominal wheel speed	140 rad/s
The closed loop bandwidth the reaction wheel	10.0 rad/s
Maximum torque produced by the reaction wheel	0.0075 N
The reaction wheel assembly is defined by the matrix transforming the wheel frames to the principal coordinate system	$\begin{bmatrix} \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} \\ \sqrt{\frac{2}{3}} & -\sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{2}{3}} & \sqrt{\frac{2}{3}} \end{bmatrix}$
Proportional control gain	$\boldsymbol{K}_p = 2.16 \times 10^{-3}\boldsymbol{E}$
Derivative control gain	$\boldsymbol{K}_d = -0.14\boldsymbol{E}$

3.4. Simulation validation

The control algorithm is validated by the simulation test performed in Matlab®/Simulink® environment. The spacecraft parameters and control settings used in the study are listed in Table 1.

When choosing the control parameters, matrices $\boldsymbol{K}_d$ and $\boldsymbol{K}_p$ the following considerations are taken into account:

- maximum torque produced by the reaction wheel assembly,
- maximum allowable angular velocity of the reaction wheel,
- large $\boldsymbol{K}_p$ gain contributes to quick initialization of the spacecraft slew maneuver (fast slew maneuver),
- large $\boldsymbol{K}_d$ gain contributes to good disturbance attenuation.

Two examples of simulation tests are shown in Figs. 4 and 5. Fig. 4 depicts the test for the initial attitude $[0.5\ 0.5\ 0.5\ \ -0.5]^{\mathrm{T}}$, and the reference at the identity quaternion. It is seen that the inclination angle between the bore axis of the star camera and the Sun vector increases to 125°. Afterwards, it is reduced to 45°, which reflects the inclination angle at the reference. Fig. 5 illustrates the simulation test for the initial attitude quaternion $[0\ 0\ 1\ 0]^{\mathrm{T}}$ and the reference $[0.32\ 0.48\ 0.80\ 0.16]^{\mathrm{T}}$. The inclination between the star camera's bore axis and the sun vector increases to 90° then converges to the value at the reference.

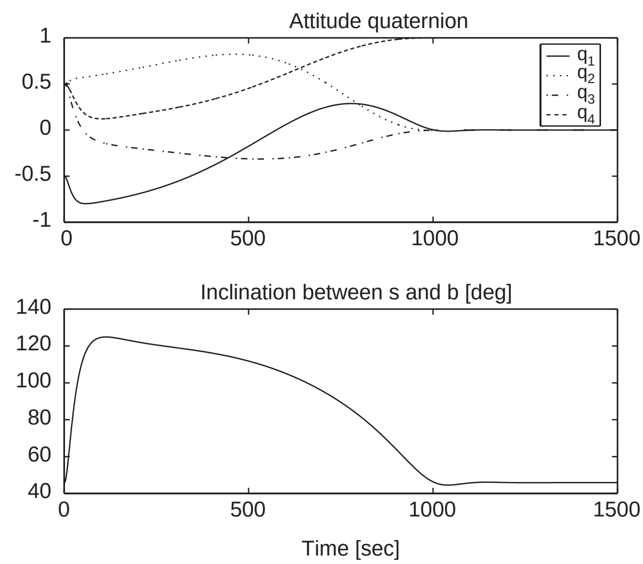

Fig. 4. Slew maneuver for the initial attitude $[0.5\ 0.5\ 0.5\ \ -0.5]^{\mathrm{T}}$, and the reference $\boldsymbol{e}$.

4. Control torque allocation

The Control torque allocation provides ability to distribute the control torque computed by the attitude controller among the reaction wheels in tetrahedron configuration.

4.1. Optimal momentum distribution problem

The problem of angular momentum distribution will be formulated and subsequently solved in this section. The problem considered is to find minimum of the function J

$$\min_{\boldsymbol{h}_w} J = \min_{\boldsymbol{h}} \|\boldsymbol{h} - \bar{\boldsymbol{h}}_w\| \tag{31}$$

subject to the constraint equation

$$\boldsymbol{D}\boldsymbol{h}_w = \boldsymbol{h}, \tag{32}$$

where $\|\cdot\|$ denotes the standard Euclidean norm, $\boldsymbol{h}_w$ means the vector of which ith component h_w^i is the angular momentum vector of ith momentum wheel, $\bar{\boldsymbol{h}}_w$ stands for the nominal value of $\boldsymbol{h}_w$. The problem (31),

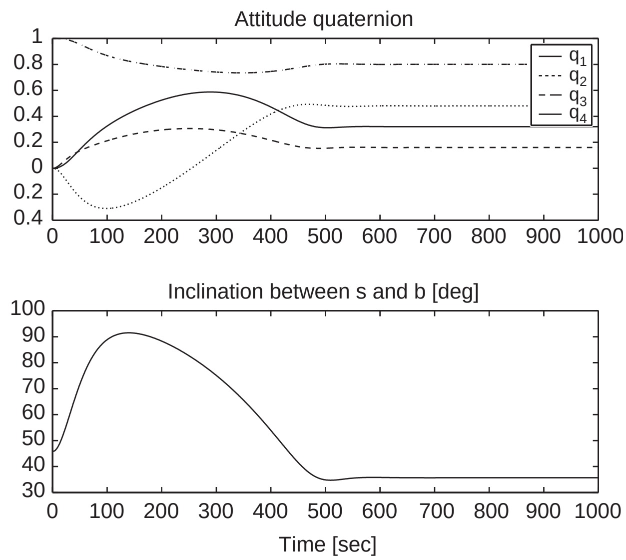

Fig. 5. Slew maneuver for initial attitude quaternion $[0\ 0\ 1\ 0]^{\mathrm{T}}$, and the reference $[0.32\ 0.48\ 0.80\ 0.16]^{\mathrm{T}}$.

(32) is denoted the optimal momentum distribution problem (OMDP).

Knowing the angular momentum of the reaction wheels $\boldsymbol{h}_w(k)$ at the time instant t_k and a constant value of the control torque $\boldsymbol{M}_c$ in the time interval $[t_k, t_{k+1}[$, the increment of the angular momentum is calculated $\Delta\boldsymbol{h}_w = \boldsymbol{M}_c T_s$, where $T_s = t_{k+1} - t_k$ means the sampling time in the discrete time implementation.

The difference between the present value of the angular momentum $\boldsymbol{h}_w(t_k)$ and the nominal value is denoted by $\Delta\boldsymbol{H}_w = \boldsymbol{h}_w(t_k) - \bar{\boldsymbol{h}}_w$. To formulate the OMDP as one of the standard static optimization problems, two vectors $\Delta\boldsymbol{L}_w$ and $\Delta\boldsymbol{L}$ are defined

$$\Delta\boldsymbol{L}_w = \Delta\boldsymbol{h}_w + \Delta\boldsymbol{H}_w,$$
$$\Delta\boldsymbol{L} = \Delta\boldsymbol{h} + \boldsymbol{D}\Delta\boldsymbol{H}_w. \tag{33}$$

Now, the OMDP is expressed as follows: Find $\Delta\boldsymbol{L}_w$ such that

$$\min_{\boldsymbol{L}_w} \|\boldsymbol{L}_w\| \tag{34}$$

subject to

$$\boldsymbol{D}\Delta\boldsymbol{L}_w = \Delta\boldsymbol{L}. \tag{35}$$

The optimization problem (34) and (35) has the solution; Griffel (1989)

$$\Delta\boldsymbol{L}_w = \boldsymbol{D}^{\mathrm{R}}\Delta\boldsymbol{L}, \tag{36}$$

where $\boldsymbol{D}^{\mathrm{R}} = \boldsymbol{D}^{\mathrm{T}}(\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}})^{-1}$, and it means the right pseudo-inverse of $\boldsymbol{D}$.

After substitution of Eq. (33) into Eq. (36), the solution to the OMDP (31), (32) is

$$\Delta\boldsymbol{h}_w = \boldsymbol{D}^{\mathrm{R}}\Delta\boldsymbol{h} - (\boldsymbol{E} - \boldsymbol{D}^{\mathrm{R}}\boldsymbol{D})\Delta\boldsymbol{H}_w. \tag{37}$$

Eq. (37) has an elegant geometric interpretation. The image of $\boldsymbol{D}^{\mathrm{R}}$ coincides with the image of $\boldsymbol{D}^{\mathrm{T}}$ and the image of $(\boldsymbol{E} - \boldsymbol{D}^{\mathrm{R}}\boldsymbol{D})$ is the kernel of $\boldsymbol{D}$. Hence the two terms on the right hand side of Eq. (37) are orthogonal. Furthermore, $\Delta\boldsymbol{h}_w$ satisfying $\min_{\boldsymbol{h}_w}\|\boldsymbol{h}_w\|$ subject to $\boldsymbol{D}\Delta\boldsymbol{h}_w = \Delta\boldsymbol{h}$ is $\Delta\boldsymbol{h}_w = \boldsymbol{D}^{\mathrm{R}}\Delta\boldsymbol{h}$. The second term in Eq. (37) is used to remove the excess of the angular momentum of each reaction wheel from its nominal value.

Finally, Eq. (37) shall be rewritten using information about the computed control torque $\boldsymbol{M}_c$ and the torque generated by the wheels $\boldsymbol{M}_w$. This can be done using an observation that the control torque is constant between samples

$$\boldsymbol{M}_w = \boldsymbol{D}^{\mathrm{R}}\boldsymbol{M}_c - (\boldsymbol{E} - \boldsymbol{D}^{\mathrm{R}}\boldsymbol{D})\quad \frac{\Delta\boldsymbol{H}_w}{T_s}. \tag{38}$$

4.2. Simulation validation

The control torque allocation is designed to keep the angular momentum of the reaction wheels near their nominal values. Fig. 6 illustrates this functionality. The slew maneuver controller is active during 2000 s. The angular momentum starts at $[-140\ -140\ -140\ -140]^{\mathrm{T}}$ Nms. The CTA distributes the control torque such that the angular momentum converges towards the nominal value $[140\,140\,140\,140]^{\mathrm{T}}$ Nms. This shows that CTA not only distributes the angular momentum such that this results in the desired control torque, but it also minimizes discrepancy between

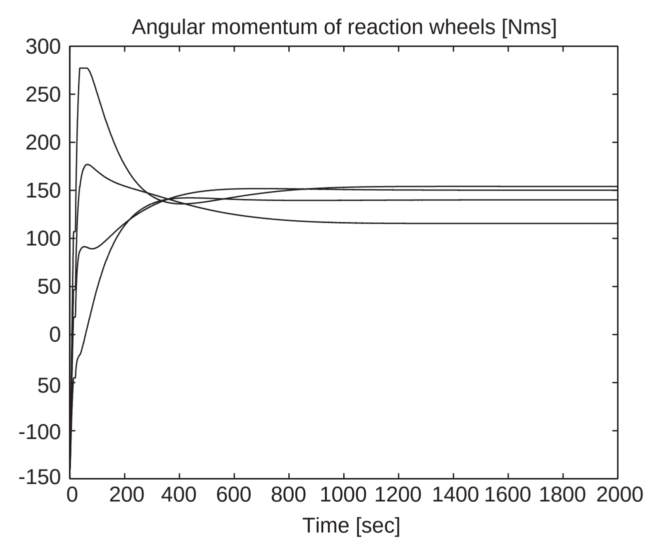

Fig. 6. The graph shows the angular momentum of four reaction wheels with the initial value of the angular momentum $[-140\ -140\ -140\ -140]^{\mathrm{T}}$ Nms. The slew maneuver controller is activated for 2000 s. The algorithm distributes the control torque such that the angular momentum of the wheels converges towards the nominal value $[140\,140\,140\,140]^{\mathrm{T}}$ Nms.

the actual and nominal values of the angular momentum for each wheel.

5. Conclusions

The slew maneuver controller was proposed for a spacecraft equipped with a star camera and 4 reaction wheels in the tetrahedron configuration. The controller development was based on the energy shaping method. The desired potential function was carefully designed using a physical insight into the nature of the problem. The controller was designed to satisfy requirement that during the maneuver the camera should never be exposed to the direct Sun light. A second task of the controller was to distribute the control torque among the reaction wheels in such a way that the resulting angular momentum of each wheel was nearest to its nominal value. A detailed simulation study showed convincing results for entire envelope of operation. The result of this work is an easily realizable controller suited for on-board implementation.

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