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Fuzzy controller for a system with uncertain load

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Abstract

In many applications of motion control, problems associated with imprecisely measured or changing load (a mass or a moment of inertia) can be a serious obstacle in the formation of satisfactory controlling systems. This barrier compels the designer to include various kinds of uncertainties in engineering solutions. The present paper deals with the time-optimal control for mechanical systems with uncertain load. A fuzzy approach is used in the design of suboptimal feedback controllers, robust with respect to the load. The methodology proposed in this work may be easily adapted to other modeling uncertainties of mechanical systems, e.g. parameters of drive or motion resistance.

Keywords: Fuzzy control; Optimal control; Robotics; Decision analysis

1. Introduction

The history of time-optimal (minimum-time) control dates back as to early as to the introduction of optimization theory in control engineering. The basic solutions for mechanical systems are well known and can be found in many textbooks (see e.g. [2, Chapter 7.2] or [4, Section 8]). However, these solutions rely on precise knowledge of the model of a plant; therefore, the performance of the standard time-optimal control is sensitive to any mismatches and uncertainties that may occur. This is the reason why many authors have recently taken up the problem of robustness for such control systems; cf. in particular classical mode control [5,15], where the current state of the art can be found in [1,3,14,16,17] and also in [7,10]. One of the main sources of uncertainty in models of mechanical systems is the load, or, to be more precise, the value of a mass or a moment of inertia. In practice, this value may be given with only the degree of precision allowed by measurement errors. Moreover, in many applications (e.g. shifting or transport tasks) this value is not subject to measurement at all, but rather is grossly estimated on the basis of an assumed value. In still other cases, the load may be variable, in tandem with the consumption of fuel or other substances used in the technological process. In the present paper, this problem has been solved by the introduction of the fuzzy type of uncertainty, which makes it possible to propose new types of control structures that take into account uncertain load, without the undue complication of a control law. Empirical tests have confirmed the satisfactory performance of the structures proposed, indicating a considerable number of advantages, especially with respect to robustness.

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2. Preliminaries and problem formulation

The dynamics of single-degree-of-freedom mechanical systems is described by the second law of Newtonian mechanics: $m\ddot{x}(t) = u(t)$, restated as

$$\dot{x}(t) = y(t), \tag{1}$$

$$\dot{y}(t) = \frac{1}{m}u(t), \tag{2}$$

where x, y are, respectively, the position and the velocity of an object, $m > 0$ is its load (mass or moment of inertia), and u is a control (either a force or a torque); let the value of the control be bounded to the interval $[-1, 1]$. The problem considered in this paper is that of a time-optimal control, i.e. minimizing the value of the cost functional $J(u) = t_T - t_0$, where t_0 and t_T are an initial time and a time to reach an assumed target state, respectively. Denote the target state as $[x_T, y_T]^T \in \mathbb{R}^2$, and suppose that $[x_+, y_+]^T$ and $[x_-, y_-]^T$ are unique solutions [8] of differential equations (1)–(2) with the condition $[x_+(0), y_+(0)]^T = [x_-(0), y_-(0)]^T = [x_T, y_T]^T$, defined on the interval $(-\infty, 0]$, and generated by the control $u \equiv +1$ or $u \equiv -1$, respectively. Moreover, consider

$$K_+ = \{[x_+(t), y_+(t)]^T \quad \text{for } t < 0\}, \tag{3}$$

$$K_- = \{[x_-(t), y_-(t)]^T \quad \text{for } t < 0\}; \tag{4}$$

therefore, these are the sets of all states which can be brought to the target $[x_T, y_T]^T$ by the control $u \equiv +1$ or -1, respectively. Let also

$$R_+ = \{[x, y]^T \text{ such that there exists } [x^*, y]^T \in K \text{ with } x > x^*\}, \tag{5}$$

$$R_- = \{[x, y]^T \text{ such that there exists } [x^*, y]^T \in K \text{ with } x < x^*\}, \tag{6}$$

where $K = K_- \cup \{[x_T, y_T]^T\} \cup K_+$ (see also Figs. 1a and 2a). The time-optimal control is then expressed by the following formula:

$$u(t) = u_r(x(t), y(t)) = \begin{cases} -1 & \text{if } [x(t), y(t)]^T \in (R_- \cup K_-), \\ 0 & \text{if } [x(t), y(t)]^T = [x_T, y_T]^T, \\ +1 & \text{if } [x(t), y(t)]^T \in (R_+ \cup K_+) \end{cases} \tag{7}$$

and the set K constitutes a switching curve. More details are found in [2, Chapter 7.2] or [4, Section 8].

In the time-optimal feedback controller equations, i.e. formulas (3)–(7), the parameter m intervenes, because it influences the form of the trajectories $[x_+, y_+]^T$, $[x_-, y_-]^T$ and therefore also the shape of the switching curve K. The analysis of the system's sensitivity to the value of that parameter (cf. also [10]), which is briefly presented below, is consequently of great importance. Thus, the value of the parameter m occurring in the object is still denoted as m; however, the value used in feedback controller equations will be marked by M; accordingly, the parameter M can be interpreted as an (indefinite) knowledge about the parameter m needed for the purpose of designing the feedback controller.

The case where the second co-ordinate of the target state is equal to zero, i.e. with $y_T = 0$, will be considered first.

If $M = m$, the control is time optimal (Fig. 1a). The state of the system is brought to the switching curve, and being permanently included in this curve hereafter, it reaches the target in a minimal and finite time.

The trajectory representative for $M < m$ is shown in Fig. 1b. As a result of the oscillations around the target, over-regulations occur in the system. The target is reached in a finite time.

Fig. 1c, however, shows the trajectories that are representative when $M > m$. After the switching curve is crossed, sliding trajectories [13] appear in the system. Here, too, the target is reached in a finite time.

In both of the last two cases, i.e. with $M \neq m$, the time to reach the target state increases from the optimal more or less proportionally to the difference between the values M and m.

The remaining case, $y_T \neq 0$, will now be presented.

If $M = m$, the control is time optimal, and the phenomena are identical as before for $y_T = 0$ (see also Fig. 2a).

When $M < m$, the trajectories occurring in the system generate cycles (Fig. 2b); the target is not reached in a finite time.

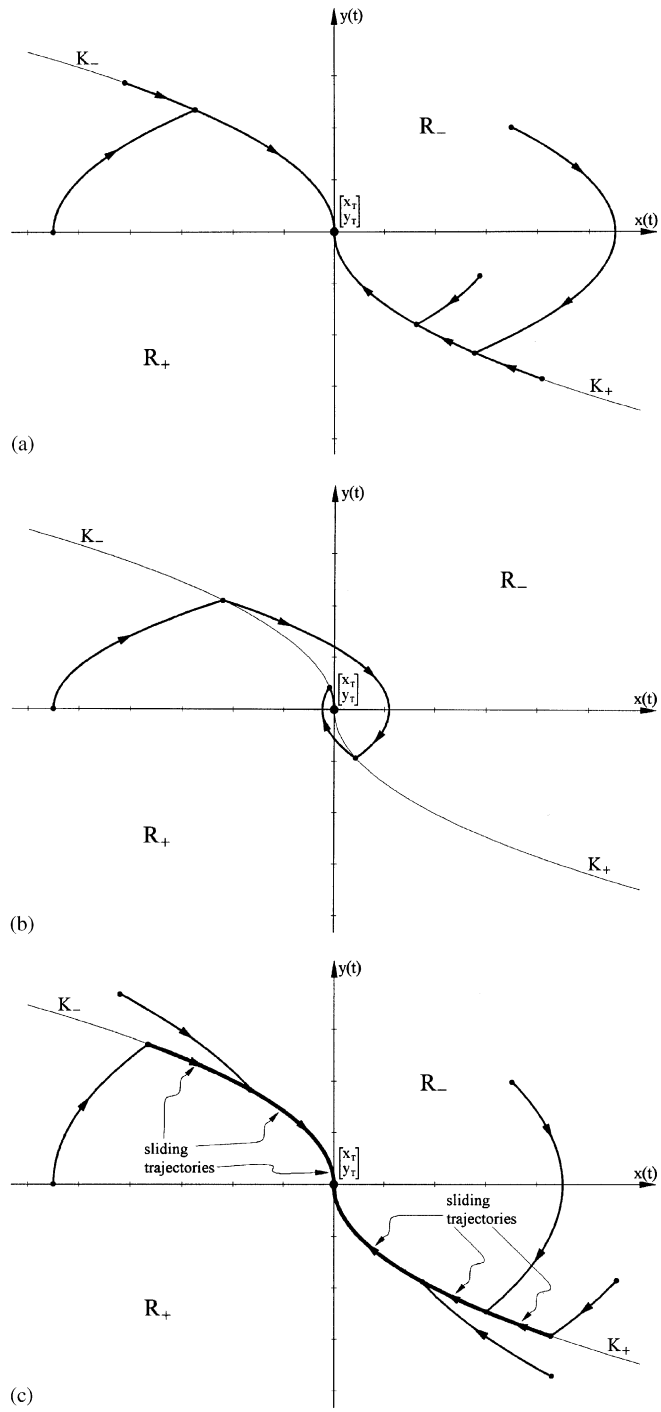
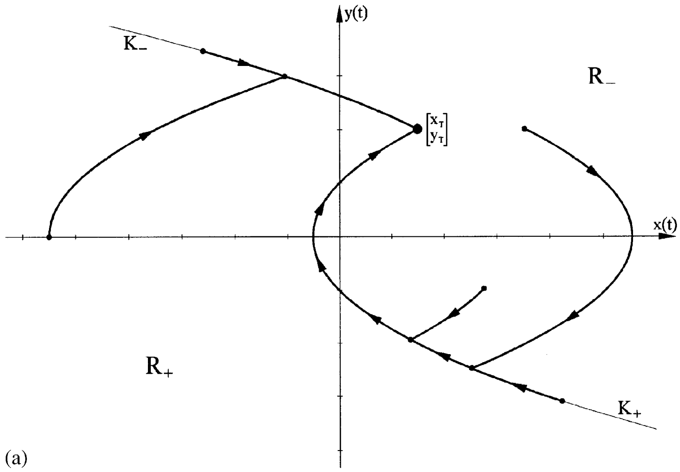
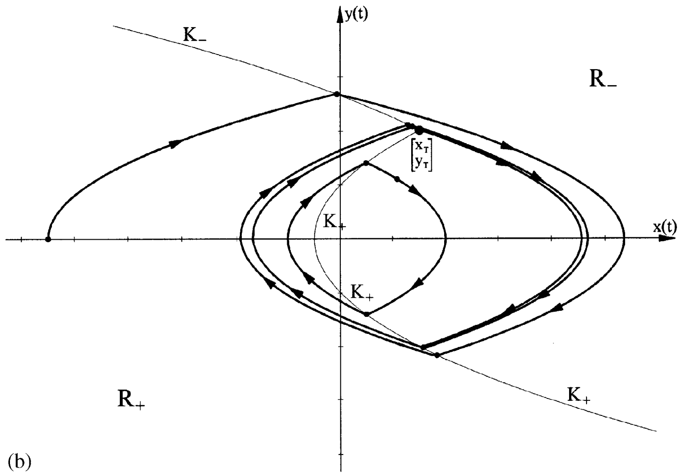


Fig. 1. The case $y_T = 0$: (a) $M = m$ (optimal control), (b) $M < m$, (c) $M > m$.





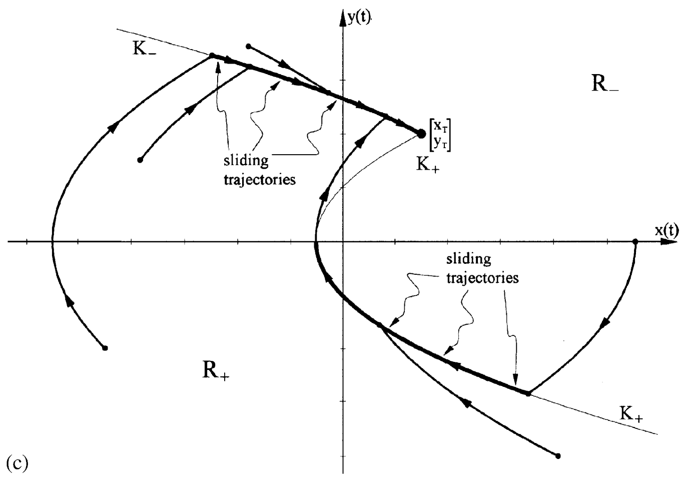


Fig. 2. The case $y_T \neq 0$: (a) $M = m$ (optimal control), (b) $M < m$, (c) $M > m$.

Finally, Fig. 2c illustrates the situation for $M > m$; even though some of the trajectories temporarily diverge from the switching curve in the part between the axis x and the target state, ultimately the target is reached in a finite time. Sliding trajectories exist on the switching curve. The time to reach the target increases as the difference $M - m$ grows.

As can be seen, a discrepancy in the correct value of the parameter m, besides increasing the time to reach the target, results in various phenomena: sliding trajectories, over-regulations, or limit cycles. Yet in many mechanical systems, sliding trajectories can have a very negative impact on the actuator life and excite vibrations in elastic transmissions, hence they should be avoided. In other applications, over-regulations may be inadmissible due to spatial limitations beyond the target, or to considerations of convenience. Finally, in the case $y_T \neq 0$, limit cycles effectively cancel the practical usability of the system. In order to construct a controlling structure eliminating these hindrances, the fuzzy approach is used in what follows. The number m will be treated as a fuzzy set M with a bounded support. By its very nature, such an approach offers the possibility to describe a complex reality with a precision that exceeds classical modeling techniques. Allowing for the degree of discomfort resulting from the uncertainty introduced into the model (note e.g. that the state of the dynamical system becomes fuzzy, too), one may obtain a feature that is essential in modern engineering: the robustness of the designed control system.

Finally, let

(A) $[x_0, y_0]^T \in \mathbb{R}^2$ and $[x_T, y_T]^T \in \mathbb{R}^2$ represent the initial and target states, respectively;

(B) M denote a fuzzy set with a support such that $\operatorname{supp}(\mathsf{M}) = [m_*, m^*] \subset (0, \infty)$;

(C) the difference equation

$$\mathsf{X}_{j+1} = \mathsf{X}_j + \mathsf{Y}_j, \tag{8}$$

$$\mathsf{Y}_{j+1} = \mathsf{Y}_j + \frac{1}{\mathsf{M}} U_j \tag{9}$$

and the initial condition

$$\begin{bmatrix} \mathsf{X}_0 \\ \mathsf{Y}_0 \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \tag{10}$$

describe the dynamics of a system with the fuzzy state $[\mathsf{X}_j, \mathsf{Y}_j]^T$, subject to the control U_j bounded to the interval $[-1, 1]$.

The goal of this paper is to design a suboptimal (with respect to time) feedback controller, whose values directly depend only on the valid state of the object, obtained by a real-time measurement process.

3. Main results

Consider fuzzy system (8)–(10). The real parameter m, natural in the second law of Newtonian mechanics (1)–(2), happens to be the fuzzy set M in the time-optimal control problem considered hereinafter. A fuzzy set naturally cannot be used directly to define a control in a real system. For this reason, some elements of fuzzy decision theory [6] will be used. Its aim is to make the optimal selection of one element from all possible decisions on the basis of a membership function. Let the following be given: a fuzzy set Z (with the membership function $\mu_{\mathsf{Z}} : \mathbb{R} \to [0, \infty)$) representing the state of reality, a non-empty set D of possible decisions, and a loss function

$$l : D \times \mathbb{R} \to \mathbb{R} \cup \{\pm\infty\}, \tag{11}$$

where the values $l(d, z)$ can be interpreted as losses occurring in the hypothetical case when the fuzzy set Z is reduced to the real number z, and the decision d has been made. Denote by $l_m : D \to \mathbb{R} \cup \{\pm\infty\}$ the minimax loss function

$$l_m(d) = \sup_{z \in \operatorname{supp}(\mathsf{Z})} l(d, z). \tag{12}$$

If additionally for every $d \in D$ the integral $\int_{\mathbb{R}} l(d, z) \mu_{\mathsf{Z}}(z)\, dz$ exists, suppose also that the Bayes loss function $l_b : D \to \mathbb{R} \cup \{\pm\infty\}$ is defined as

$$l_b(d) = \int_{\mathbb{R}} l(d, z) \mu_{\mathsf{Z}}(z)\, dz. \tag{13}$$

Every element $d_m \in D$ such that

$$l_m(d_m) = \inf_{d \in D} l_m(d) \tag{14}$$

is called a minimax decision, and analogously, every element $d_b \in D$ such that

$$l_b(d_b) = \inf_{d \in D} l_b(d) \tag{15}$$

is called a Bayes decision. The procedures for obtaining these elements are said to be minimax and Bayes rules, respectively. The main difference between the above rules appears in their interpretation. This results directly from the forms of the functions l_m and l_b: the "pessimistic" minimax rule assumes the occurrence of the most unfavorable state of reality and counteracts it, while the Bayes rule is more flexible [10].

In the problem of the time-optimal control investigated here, the parameter $M \in D \subset (0, \infty)$ used in the feedback controller equations will be treated as a decision, and the number M occurring in system (8)–(10) as the fuzzy state of reality. Loss function (11) is defined for $(M, m) \in D \times \mathbb{R}$, and its values are related to the time to reach the target, if the parameter M was used in the feedback controller equations, while hypothetically the value m is present in the object.

Once again the case $y_T = 0$ will be considered first. The following suggestions for the determination of the value of the parameter M result from the sensitivity analysis presented in the previous section.

If over-regulations can be allowed, it is worthwhile using the Bayes rule with real values for the loss function. Such a choice is possible because the determination of the parameter M value that is either less than, equal to, or greater than m allows the system state to be brought to the target in a finite time. (However, this time increases approximately proportionally to the difference between the values M and m.)

If over-regulations are not allowed, this determination needs to be carried out on the basis of the minimax rule, assuming infinite values of the loss function for $M < m$. This enables over-regulations to be avoided, because they occur only if $M < m$.

Let now $y_T \neq 0$. The value of the parameter M should be determined using the minimax rule with infinite values of the loss function for $M < m$. This choice is made in order to avoid the generation of inadmissible limit cycles, which appear when $M < m$. If, however, this value is greater than m, the state of the system is brought to the target in a finite time. (Note that in the case $y_T \neq 0$, over-regulations cannot be avoided; see Figs. 2a–c.)

Suppose, as an example, that loss function (11) is described by the following formula:

$$l(M, m) = \begin{cases} -p(M - m) & \text{if } M - m \leqslant 0, \\ q(M - m) & \text{if } M - m \geqslant 0, \end{cases} \tag{16}$$

where $p, q \in \mathbb{R}_+ \cup \{\infty\}$, but only one of them can be infinite; in this case, let $\infty \cdot 0 = 0$.

According to assumption (B) made in Section 2 and the above considerations, it is accepted that $D = [m_*, m^*]$.

With a fixed value for the parameter M, the following results from the definition of the minimax loss function (12):

$$l_m(M) = \max(\{-p(M - m^*), q(M - m_*)\}). \tag{17}$$

If $p = \infty$, the infimum of the function l_m on the set D is realized by

$$M = m^*. \tag{18}$$

The above value constitutes the desired minimax decision with infinite values of loss function (16) for $M < m$.

In turn, since for the positive numbers p and q the inequality $0 < p/(p+q) < 1$ is true, there exists a real number such that the following equation with the argument M:

$$\int_{m_*}^{M} \mu_{\mathsf{M}}(m)\,dm = \frac{p}{p+q} \int_{m_*}^{m^*} \mu_{\mathsf{M}}(m)\,dm \tag{19}$$

is fulfilled. Denote this number as $\hat{M}$ and observe that it is unique because—as results from assumption (B) formulated in Section 2—the function μ_{M} is positive almost everywhere in the interval $[m_*, m^*]$. Now, by inserting formula (16) into definition (13) one obtains

$$\begin{aligned} l_b(M) = & \int_{m_*}^{m^*} [q(M - m)\chi_{(-\infty, M]}(m) \\ & - p(M - m)\chi_{[M, \infty)}(m)]\mu_{\mathsf{M}}(m)\,dm \\ = & (p+q) \int_{m_*}^{m^*} (M - m) \\ & \times \left[\left(1 - \frac{p}{p+q}\right) \chi_{(-\infty, M]}(m) \right. \end{aligned}$$

$$- \frac{p}{p+q}\chi_{[M,\infty)}(m)\Bigg]\,\mu_{\mathrm{M}}(m)\,\mathrm{d}m$$

$$= (p+q)\int_{m_*}^{m^*}(M-m)$$

$$\times \left[\chi_{(-\infty,M]}(m) - \frac{p}{p+q}\right]\mu_{\mathrm{M}}(m)\,\mathrm{d}m, \quad (20)$$

where χ_A means the characteristic function of the set A. In particular,

$$l_{\mathrm{b}}(\hat{M}) = (p+q)\int_{m_*}^{m^*}(\hat{M}-m)$$

$$\times \left[\chi_{(-\infty,\hat{M}]}(m) - \frac{p}{p+q}\right]\mu_{\mathrm{M}}(m)\,\mathrm{d}m$$

$$= (p+q)(\hat{M}-M)$$

$$\times \int_{m_*}^{m^*}\left[\chi_{(-\infty,\hat{M}]}(m) - \frac{p}{p+q}\right]\mu_{\mathrm{M}}(m)\,\mathrm{d}m$$

$$+ (p+q)\int_{m_*}^{m^*}(M-m)$$

$$\times \left[\chi_{(-\infty,\hat{M}]}(m) - \frac{p}{p+q}\right]\mu_{\mathrm{M}}(m)\,\mathrm{d}m; \quad (21)$$

therefore, since $\hat{M}$ fulfills Eq. (19):

$$l_{\mathrm{b}}(\hat{M}) = (p+q)\int_{m_*}^{m^*}(M-m)$$

$$\times \left[\chi_{(-\infty,\hat{M}]}(m) - \frac{p}{p+q}\right]\mu_{\mathrm{M}}(m)\,\mathrm{d}m. \quad (22)$$

The combination of dependencies (20) and (22) yields

$$l_{\mathrm{b}}(M) - l_{\mathrm{b}}(\hat{M})$$

$$= (p+q)\int_{m_*}^{m^*}(M-m)[\chi_{(-\infty,M]}(m)$$

$$- \chi_{(-\infty,\hat{M}]}(m)]\mu_{\mathrm{M}}(m)\,\mathrm{d}m$$

$$= (p+q)\int_{m_*}^{m^*}|M-m|[\chi_{(-\infty,M]}(m)$$

$$- \chi_{[M,\infty)}(m)][\chi_{(-\infty,M]}(m)$$

$$- \chi_{(-\infty,\hat{M}]}(m)]\mu_{\mathrm{M}}(m)\,\mathrm{d}m \quad (23)$$

and finally, elementary manipulations on the characteristic functions give

$$l_{\mathrm{b}}(M) - l_{\mathrm{b}}(\hat{M}) = (p+q)\int_{m_*}^{m^*}|M-m|[\chi_{[M,\hat{M}]}(m)$$

$$+ \chi_{[\hat{M},M]}(m)]\mu_{\mathrm{M}}(m)\,\mathrm{d}m \geqslant 0. \quad (24)$$

The above dependence proves that the Bayes loss function l_{b} has the global minimum for $M = \hat{M}$. Finally, the value M that fulfills Eq. (19) constitutes the desired Bayes decision with real values of loss function (16). To calculate this value one can use the kernel technique [11], according to the procedure presented in [12], where neural networks have also been applied.

To summarize, for the loss function defined by formula (16), if the values of the parameter M are to be determined according to the minimax rule with infinite values of the loss function for $M < m$ or the Bayes rule with real values of this function, then they can be obtained from dependencies (18) and (19), respectively.

Having the value M, the feedback controller equations can be calculated. Thus, the equations of the switching curve K take on the form

$$x = -\frac{M}{2}(y^2 - y_{\mathrm{T}}^2) + x_{\mathrm{T}} \quad \text{for } y \in [y_{\mathrm{T}}, \infty), \quad (25)$$

$$x = \frac{M}{2}(y^2 - y_{\mathrm{T}}^2) + x_{\mathrm{T}} \quad \text{for } y \in (-\infty, y_{\mathrm{T}}]. \quad (26)$$

Formula (25) defines the set K_-, whereas dependence (26) describes the set K_+. The sets R_- and R_+ form areas resulting from the section of the plane $\mathbb{R}^2$ by the curve K, according to formulas (5) and (6). For the sets K_-, K_+, R_-, R_+ obtained in this way, the value of the control is simply defined by the equation

$$U_j = \begin{cases} -1 & \text{if } [X_j, Y_j]^{\mathrm{T}} \in (R_- \cup K_-), \\ 0 & \text{if } [X_j, Y_j]^{\mathrm{T}} = [x_{\mathrm{T}}, y_{\mathrm{T}}]^{\mathrm{T}}, \\ +1 & \text{if } [X_j, Y_j]^{\mathrm{T}} \in (R_+ \cup K_+), \end{cases} \quad (27)$$

where $[X_j, Y_j]^{\mathrm{T}}$ means the object state, obtained by a real-time measurement process in the moment j. Fig. 3 provide an illustration of the control structure worked out here and the trajectories it generates.

The control designed above may lead to chattering, i.e. frequent switchings between the two values +1

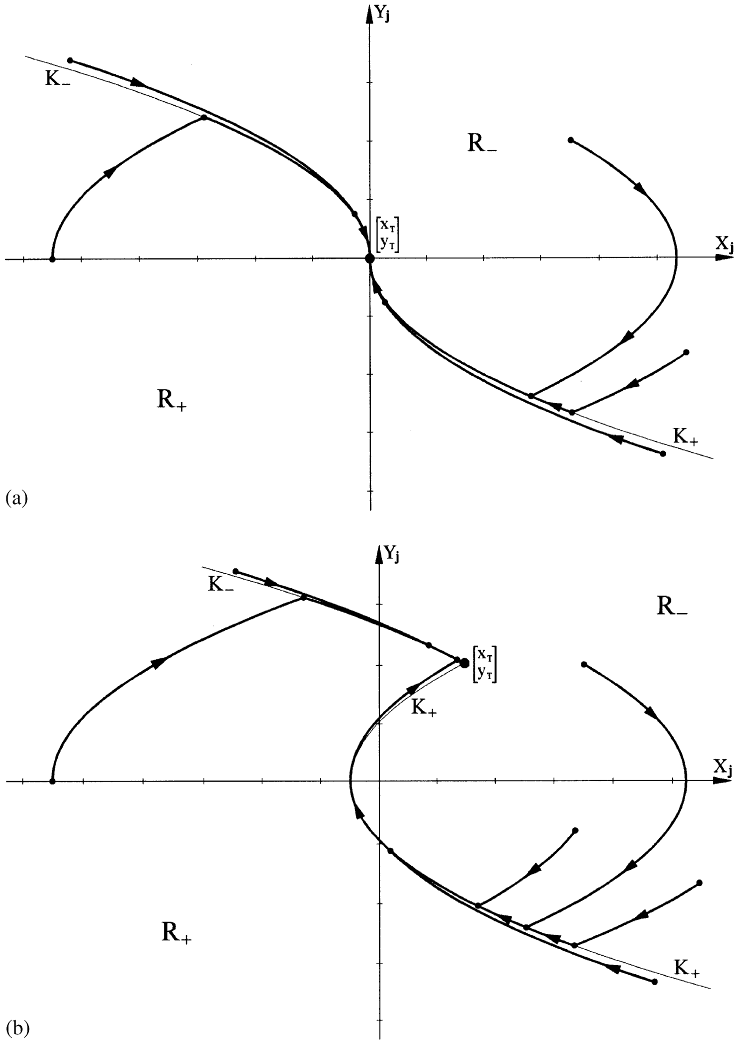


Fig. 3. Fuzzy feedback controller (26) and trajectories: (a) $y_T = 0$, (b) $y_T \neq 0$.

and -1 along sliding trajectories. As mentioned earlier, in mechanical systems such a phenomenon should be avoided, since it can have a negative impact on the endurance of a device and user comfort. Under the condition that the control may take any value in the interval $[-1, 1]$, this goal can be obtained by substituting a modified control law, rendered continuous instead of discontinuous (27); cf. [10].

As before, the case $y_T = 0$ will be considered first. Initially, the next parameter $M^{\sim}$ can be introduced, in addition to the constant M used so far, with the condition $M < M^{\sim}$. Apart from the sets K_- and K_+ defined by formulas (3) and (4) for the parameter m (or more precisely M), let similar sets $K_-^{\sim}$ and $K_+^{\sim}$ be given for the constant $M^{\sim}$ (see Fig. 4a). Defining, moreover,

$$Q_- = \{[x, y]^T \text{ such that there exists } [x^*, y]^T \in K_-^{\sim} \text{ and } [x^{**}, y]^T \in K_- \text{ with } x^* \leqslant x \leqslant x^{**}\}, \tag{28}$$

$$Q_+ = \{[x,y]^{\mathrm{T}} \text{ such that there exists } [x^*,y]^{\mathrm{T}} \in K_+ \text{ and } [x^{**},y]^{\mathrm{T}} \in K_+^{\sim} \text{ with } x^* \leqslant x \leqslant x^{**}\}, \tag{29}$$

$$Q = Q_- \cup \{[x_{\mathrm{T}}, y_{\mathrm{T}}]^{\mathrm{T}}\} \cup Q_+ \tag{30}$$

and slightly altering formulas (5) and (6) to

$$R_+ = \{[x,y]^{\mathrm{T}} \in \mathbb{R}^2 \backslash Q \text{ such that there exists } [x^*,y]^{\mathrm{T}} \in Q \text{ with } x < x^*\}, \tag{31}$$

$$R_- = \{[x,y]^{\mathrm{T}} \in \mathbb{R}^2 \backslash Q \text{ such that there exists } [x^*,y]^{\mathrm{T}} \in Q \text{ with } x > x^*\}, \tag{32}$$

the time-optimal control can now be given by

$$U_j = \begin{cases} -1 & \text{if } [X_j, Y_j]^{\mathrm{T}} \in R_-, \\ -z(-X_j, -Y_j) & \text{if } [X_j, Y_j]^{\mathrm{T}} \in Q_-, \\ 0 & \text{if } [X_j, Y_j]^{\mathrm{T}} = [x_{\mathrm{T}}, y_{\mathrm{T}}]^{\mathrm{T}}, \\ z(X_j, Y_j) & \text{if } [X_j, Y_j]^{\mathrm{T}} \in Q_+, \\ +1 & \text{if } [X_j, Y_j]^{\mathrm{T}} \in R_+, \end{cases} \tag{33}$$

where the function $z: \mathbb{R}^2 \to \mathbb{R}$ is continuously and strictly increasing from the value -1 on the sets K_- and $K_+^{\sim}$ to the value 1 on the sets $K_-^{\sim}$ and K_+. A suitable value for the parameter $M^{\sim}$ can be determined heuristically; in general, the difference $M^{\sim} - M$ should be proportional to the delay in the system. The trajectories generated by control (33) are shown in Fig. 4a for $y_{\mathrm{T}} = 0$. Thus, this control law constitutes the continuated variant of control (27), which is of the "bang–bang" type.

Now, the second case $y_{\mathrm{T}} \neq 0$ will be elaborated. As previously, in addition to the parameter M defined earlier, the further constant $M^{\sim}$ must be determined heuristically according to the delay in the system, subject to the condition $M < M^{\sim}$. The concept introduced in the preceding paragraph, expressed by formula (33), should be applied here twice in a natural way. An illustration of the control structure obtained, along with the trajectories it generates, is provided in Fig. 4b

The correct functioning of the suboptimal structures described in this paper has been verified experimentally using numerical simulation programs. The object was a mechanical system (1)–(2) with a load whose initial value was unknown (defined at random) and could change—likewise at random—with the passage of time. Typical results obtained for control structures (27) and (33) elaborated in this paper are shown in Figs. 3 and 4, respectively. These results confirmed the correctness of the theoretical considerations presented earlier, providing additional information regarding the properties of the proposed control system. Two general observations can be formulated. The first is that shorter times to reach the target were achieved in those situations where it proved possible to reduce a priori the assumed support of the number m, i.e. when the given difference $m^* - m_*$ diminished, and so the uncertainty of the load was reduced. The second is that fulfilling the additional requirements, e.g. eliminating over-regulations or sliding trajectories (by continuating structure (27)–(33)), results in a certain increase in the time to reach the target set, and thus has a somewhat deleterious effect on the basic operating goal: time-optimality. It can easily be deduced, however, that both of these observations are fully justified theoretically. The reduction of uncertainty as well as the worsening of results with respect to the basic goal, when one wishes to achieve other collateral goals in addition, are natural and obvious features, and not just in engineering.

Thus, the application of continuated structure (33) made it possible to eliminate sliding trajectories, albeit at a cost of 1–3% increase in the time to reach the target, as against the discontinuous one (27). In the case $y_{\mathrm{T}} = 0$, if it is assumed that over-regulations are undesirable, then they did not occur in the controlled object, though the times to reach the target were also up to 3% greater than those obtained without this condition.

Finally, in the case $y_{\mathrm{T}} \neq 0$—in accordance with requirements—limit cycles definitively did not appear.

In conclusion, it should be strongly emphasized that the control structures presented in this paper turn out to be only slightly sensitive to the inaccuracy resulting from identification. Such robustness should be

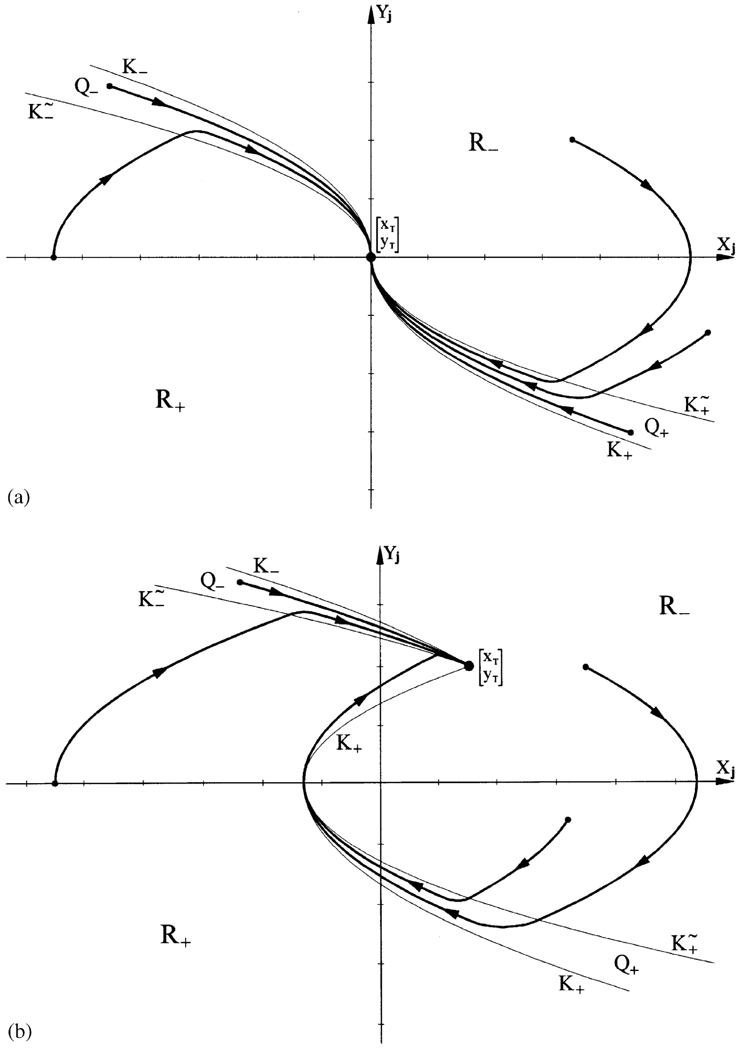


Fig. 4. Fuzzy feedback controller (32) and trajectories: (a) $y_T = 0$, (b) $y_T \neq 0$.

emphasized as a very valuable property of uncertain, especially fuzzy, control systems [9,10].

4. Generalizations

In Section 2, the problem addressed in this paper has been formulated in a fundamental version for the sake of clarity in the investigations. However, the material presented here allows for the easy introduction of generalizations to forms that are frequently used in engineering applications.

First, let u introduced in Eqs. (1) and (2) mean the moment obtained from the drive, which is treated here as an inertial element with the constant T, i.e.

$$\dot{u}(t) = -\frac{1}{T}\,u(t) + v(t), \tag{34}$$

where v is a bounded control; for the engineering basics and interpretation, see e.g. [2, Chapter 7.4]. If the number T is treated as the fuzzy set T, the concept of the feedback controller presented here can easily be generalized to a system constructed in this way. An analysis of sensitivity to the value of the parameter T produces results similar to those presented in Section 2: namely, an overly large value used for the feedback controller equations implies sliding trajectories, while too small a value generates limit cycles. Of course, due to the increase in the dimension of the vector state to 3, the switching curve crosses into the switching surface, and the analysis becomes much more complex, but the basic principles remain unchanged in terms of the fundamental concept presented earlier in this paper.

Once again, let the basis for considerations constitute the fundamental system described at the beginning of Section 2. If it is supplemented with the discontinuous model of motion resistance $-w\,\mathrm{sgn}(\dot{x}(t))$, i.e.

$$m\ddot{x}(t) = u(t) - w\,\mathrm{sgn}(\dot{x}(t)), \tag{35}$$

where $w \in [0, 1)$, then under- or overestimating the value of the parameter w will have the similar effect as raising or lowering the parameter m. In this case, however, the engineering interpretation somewhat exceeds the strict mathematical formalization presented to this point. The parameter w introduced above, in fact, reflects the variety of physical phenomena, reduced to a single constant due to the necessity to simplify the model. Then the issue here consists not so much in approaching the unknown real value (since no such thing exists), as in specifying the best possible characterization of these phenomena using a fuzzy number W. This issue is discussed more fully in [10]. It proved possible to transfer directly many of the elements introduced there to the material presented in this paper.

With respect to these aspects also, the correct functioning has been checked numerically, providing conclusions identical to those described at the end of the previous section. The application of the concept presented in this paper simultaneously to different causes of uncertainty (i.e. load, inertia of a drive, motion resistance) did not hinder the operation of the system: on the contrary, over- and underestimations of different parameters showed a tendency to mutual compensation. In this situation, the increase in the time to reach the target set was significantly less than the simple sum of such increases resulting from the uncertainty of particular factors.

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