Image Restoration in the presence of Poisson-Gaussian noise

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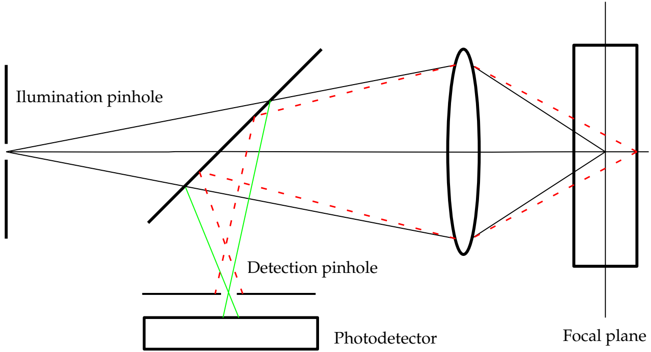
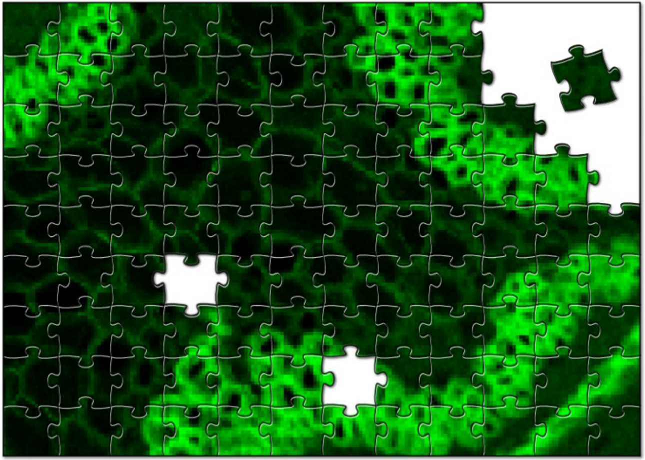
Principle of confocal imaging system



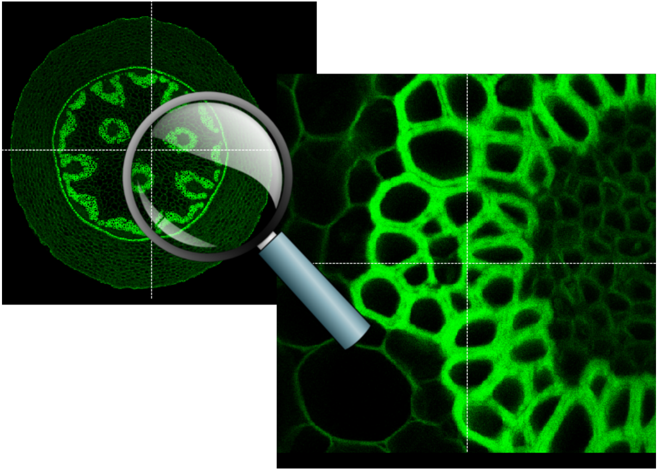
- *Much of the out-of-focus light eliminated from detection*
- *High optical resolution (sample depth direction)*
- *Image acquired point-by-point (small number of detectable photons: down to 8-12 photons per highest intensity pixel)*
- *Application area: biomedical research*

Seeing the wood for the trees



Confocal microscopy

Seeing the wood for the trees



Confocal macroscopy

Challenges

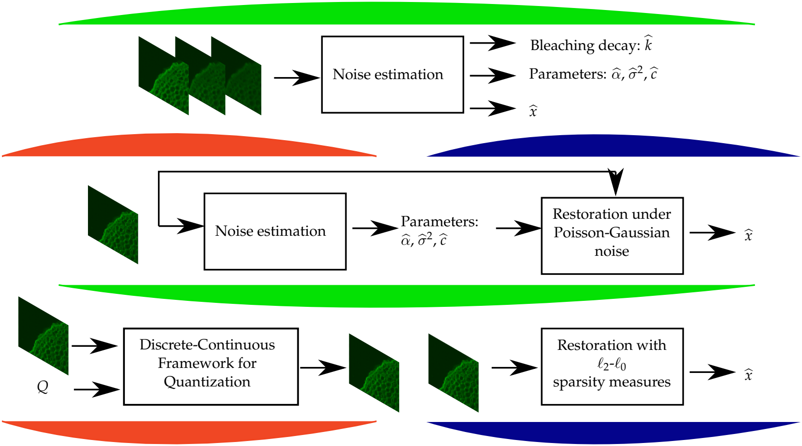
Confocal macroscopy:

- *Determine characteristics of the noise and point spread function (preferably, based on measured data)*
- *Improve statistical description of images: models incorporating richer prior knowledge*
- *Propose image deconvolution tools*

Algorithms:

- *Propose image restoration formulations taking into account image properties and noise model*
  - *Poisson-Gauss noise*
  - *Convex and non-convex regularizations*
- *Find ways to optimize these formulations exactly and efficiently*
  - *Discrete, continuous and combined approaches*
  - *Primal-dual algorithms*
- *Find ways to estimate noise parameters from available data (From time series, From single images)*

Overview



Problem: Tools related to Poisson-Gaussian noise
Problem: Discrete-continuous optimization: All variables take value from an unknown discrete set of known cardinality (inclusion of a given continuous set)
Problem: Restoration with both convex and non-convex sparsity measures

Publications

Journal papers (3):

- *Identification of Poisson-Gaussian noise parameters (IEEE TSP 2013 - in revision)*
- $\ell_2 - \ell_0$ *functions as sparsity measures (SIAM Journal on Imaging Science 2012)*
- *Discrete-continuous optimization (JMIV 2011)*

International IEEE conferences (7):

- *Identification of Poisson-Gaussian noise parameters (EUSIPCO 2011, ISBI 2012)*
- *Restoration of data corrupted with Poisson-Gaussian noise (ICASSP 2012)*
- $\ell_2 - \ell_0$ *functions as a sparsity measures (EMMCVPR 2011, ICIP 2011)*
- *Discrete-continuous optimization (ICIP 2010)*
- *Fast Recursive Ensemble Convolution of Haar-like Features (CVPR 2012)*

CONTENTS

POISSON-GAUSSIAN NOISE

Physical acquisition process:

- *Various noise sources*
  - *Some are signal dependent, e.g. photon noise*
  - *Some are signal independent, e.g. electronic noise, thermal noise etc.*
- *Too complex for either Poisson or additive Gaussian noise models*
- *Next simplest model: sum of Poisson and Gaussian models for signal dependent and signal independent components, respectively.*
- *A recent topic of interest in the literature [Snyder* et al. *1993] [Delpretti* et al. *2008] [Benvenuto* et al. *2008] [Luisier* et al. *2011] [Gil-Rodrigo* et al. *2011] [Chakrabarti and Zickler 2012] [Li* et al. *2012]*

Imaging related application areas:

- *Biology (Fluorescence imaging systems)*
- *Astronomy*
- *CCD camera imaging*

Degradation model

$$y = \alpha \; z(x) + w$$

Observations

$y \in \mathcal{Y}$
$y = (y_i)_{i \in \mathbb{Y}}$

[Zhang '07] [Boulanger *et al.* '08] [Delpretti *et al.* '08] [Luisier *et al.* '11]

DEGRADATION MODEL

Poisson noise

$x \in \mathcal{X}$ - original signal
$x = (x_i)_{i \in \mathbb{X}} \quad x_i \in [0, +\infty) \; Z_i(x) \sim \mathcal{P}([Hx]_i))$
$H : \mathcal{X} \mapsto \mathcal{Y}$ - blur
$z(x) = \left(z_i(x)\right)_{i \in \mathbb{X}}$ - realization of $\left(Z_i(x)\right)_{i \in \mathbb{X}}$
$\alpha \in (0, +\infty)$ - scale parameter

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$$y = \alpha \, z(x) + w$$

Observations

$y \in \mathcal{Y}$

$y = (y_i)_{i\in\mathbb{Y}}$

White Gaussian noise

$W_i \sim \mathcal{N}(c, \sigma^2)$

$w = (w_i)_{i\in\mathbb{X}}$ - realization of $\big(W_i\big)_{i\in\mathbb{X}}$

$c \in \mathbb{R}$ - mean $\quad \sigma^2$ - variance

[Zhang '07] [Boulanger *et al.* '08] [Delpretti *et al.* '08] [Luisier *et al.* '11]

RESTORING DATA CORRUPTED BY POISSON-GAUSSIAN NOISE

Scaled gradient method [Benvenuto *et al.* 2008]

Gradient of neg-log likelihood approximated by:

$$\exp\left(-\frac{1+2([Hx+c]_i)-y_i}{2([Hx+c]_i)+\sigma^2}\right)$$

Augmented Lagrangian method [Chakrabarti and Zickler 2012]

Neg-log likelihood approximated by:

$$\sum_{i\in\mathbb{X}}[Hx]_i+\sigma^2-\left(y_i+\sigma^2\right)\log\left([Hx]_i+\sigma^2\right)$$

Alternating-minimization alg. [Gil-Rodrigo *et al.* 2011]

Neg-log likelihood approximated by:

$$\frac{1}{2}\left\|\left(D_{\tilde{\sigma}^2}(Hx)\right)^{-1/2}(y-Hx)\right\|_2^2$$

Covariance matrix $D_{\tilde{\sigma}^2}$ depends on noise parameters

Augmented Lagrangian method [Li *et al.* 2012]

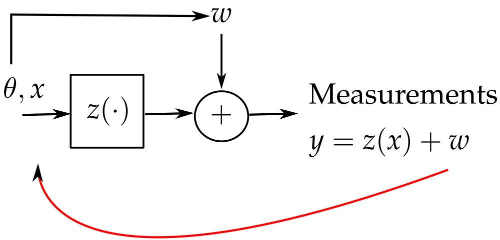
Neg-log likelihood approximated by:

$$\frac{1}{2}\left\|\left(\tilde{\sigma}^2 I+(Hx)\right)^{-1/2}(y-Hx)\right\|_2^2$$

Noise parameters θ are assumed to be known $\Rightarrow$ We need to identify them

Noise identification

IDENTIFICATION STRATEGIES



What strategy to find vector of unknown noise parameters θ?

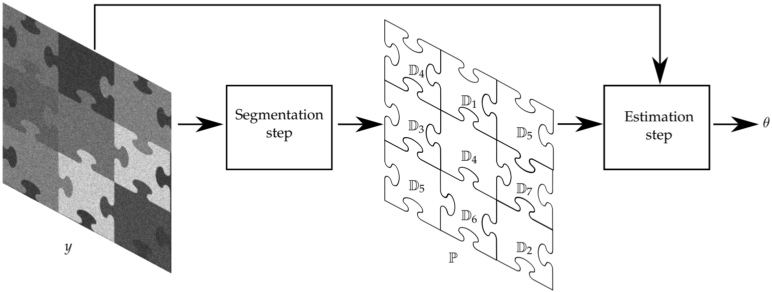
Multiple image noise identification

Difficulty: Nonstationarity of signal in time due to the bleaching process. The simplest model of bleaching: exponential decay of signal in time

Single image noise identification

Difficulty: Signal dependent noise $\Rightarrow$ One needs to find homogeneous regions of the image where the noise can be considered as stationary

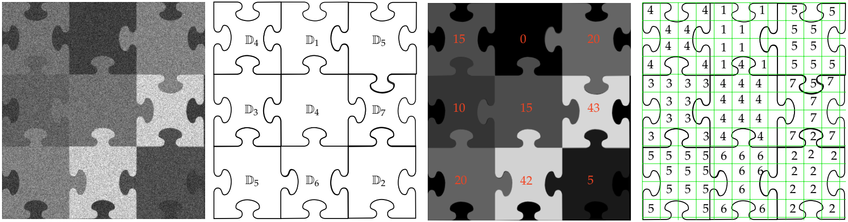
SINGLE IMAGE NOISE IDENTIFICATION - SIGNAL DEPENDENT NOISE



Existing methods ([Foi et al. 2008],[Li et al. 2010], [Paul et al. 2010], ...) - Two step approach

1. First step returns partitions $\mathbb{P} = (\mathbb{D}_k)_{1 \leq k \leq K}$ of the image support $\mathbb{X}$
2. Second step returns vector of unknown noise parameters θ

POISSON-GAUSSIAN CASE



y $\qquad$ $\mathbb{P}$ $\qquad$ $x_{i_{\mathbb{P}},u} \mid u = [0\ 5\ 10\ 15\ 20\ 42\ 43]^\top$ $\qquad$ $i_{\mathbb{P}}$

Probability density function of Y

$\forall y = \big(y(\mathbf{s})\big)_{\mathbf{s}\in\mathbb{X}} \in \mathbb{R}^{N\times M}$

$$p_Y(y; i_{\mathbb{P}}, \theta) = \prod_{k=1}^{K} \prod_{\mathbf{s}\in\mathbb{D}_k} \left(\sum_{q_{\mathbf{s}}=0}^{+\infty} \frac{e^{-u_k}\,(u_k)^{q_{\mathbf{s}}}}{q_{\mathbf{s}}!} \frac{e^{-\frac{(y(\mathbf{s})-\alpha q_{\mathbf{s}}-c)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} \right)$$

K - number of distinct intensity values , $u = [u_1, \ldots, u_K]^\top \in \mathbb{R}^K$ - codebook

label image $\big(i_{\mathbb{P}}(\mathbf{s})\big)_{\mathbf{s}\in\mathbb{X}} \in \mathbb{I} = \{1, \ldots, K\}^{N\times M}$

Problem formulation

Problem

Find $\theta = (c, \sigma^2, \alpha, u)$ such that
$\theta \in \mathcal{T} = \mathbb{R} \times (0, +\infty)^2 \times C$

♣

MAP

$$\underset{(i_{\mathbb{P}},\theta)\in\mathbb{I}\times\mathcal{T}}{\text{minimize}} \quad \Phi(\theta, i_{\mathbb{P}}, y) + \rho(i_{\mathbb{P}})$$

Where

ρ - regularization function
Φ - data fidelity term
$\Phi(\theta, i_{\mathbb{P}}, y) = -\log\left(p_Y(y; i_{\mathbb{P}}, \theta)\right)$
C - closed convex subset of $\mathbb{R}^K$

Discrete-continuous optimization framework

Fix $K \in \mathbb{N}^*$ and $\theta^{(0)} \in \mathcal{T}$

For $\ell = 0, 1, \ldots$

$$i_{\mathbb{P}}^{(\ell)} \in \underset{i_{\mathbb{P}} \in \mathbb{I}}{\operatorname{argmin}} \ \Phi(\theta^{(\ell)}, i_{\mathbb{P}}, y) + \rho(i_{\mathbb{P}})$$

$$\theta^{(\ell+1)} \in \underset{\theta \in \mathcal{T}}{\operatorname{argmin}} \ \Phi(\theta, i_{\mathbb{P}}^{(\ell)}, y)$$

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- Convex optimization step

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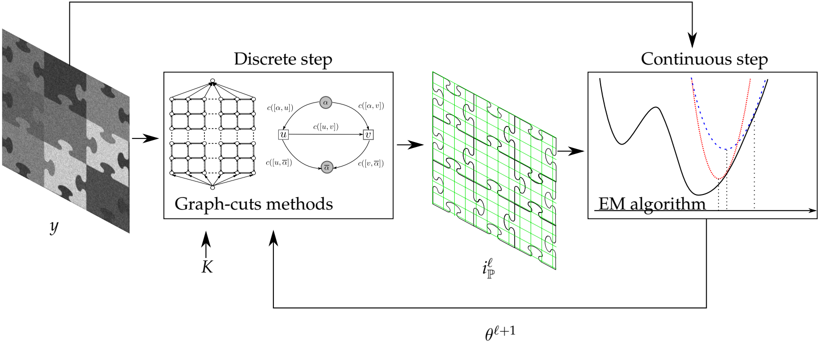
- Convex optimization step
- Combinatorial optimization step

Iterative θ estimation over image segments

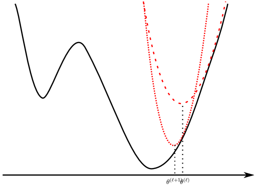


Assumption: number of distinct intensity K is assumed to be known

Sequence: $(i_{\mathcal{P}}^{(\ell)}, \theta^{(\ell+1)})_{\ell \in \mathbb{N}}$ generated such that $\left(\Phi(\theta^{(\ell+1)}, i_{\mathbb{P}}^{(\ell)}, y) + \rho(i_{\mathbb{P}}^{(\ell)})\right)_{\ell \in \mathbb{N}}$ is a convergent decaying sequence.

Regularization: efficient solution for particular choice of ρ using graph-cuts

CONTINUOUS STEP: EM ALGORITHM



Q - missing data

Majorant function - Conditional expectation

Step 1: Initialization

$\theta^{(1)} \leftarrow \left(c^{(1)}, (\sigma^2)^{(1)}, \alpha^{(1)}, u^{(1)}\right)$

Step 2: EM Algorithm

for $\ell \leftarrow 1$ to L **do**

Expectation step

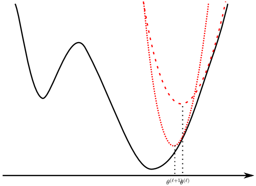
$$\Phi(\theta \mid \theta^{(\ell)}) = \mathsf{E}_{Q|Y=y,\theta^{(\ell)}}[\ln p_{Y,Q}(Y, Q \mid \theta)]$$

Maximization step

$$\theta^{(\ell+1)} = \underset{\theta \in \mathcal{T}}{\operatorname{argmin}} \; -\Phi(\theta \mid \theta^{(\ell)})$$

end for

Continuous step: EM Algorithm



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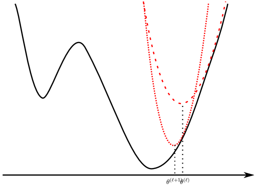

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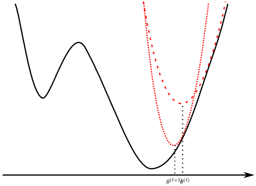

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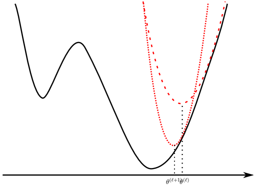

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end for

EM Maximization step

Update of the quantizations levels: $u^{(\ell+1)} \in \underset{u \in C}{\operatorname{argmin}} \, \vartheta(u)$

where $\vartheta(u) = \sum_{k=1}^{K} \vartheta_k(u_k)$, and, for every $k \in \{1, \ldots, K\}$,

$$\vartheta_k(u_k) = \operatorname{card}(\mathbb{D}_k^{(\ell)}) u_k - \ln u_k \sum_{\mathbf{s} \in \mathbb{D}_k^{(\ell)}} \mathsf{E}_{Q|R=r,\theta^{(\ell)}}[Q(\mathbf{s})]$$

Two cases:

- $C = \mathbb{R}^K$: closed form solution
- C imposes total order constraint: proximal primal-dual algorithm

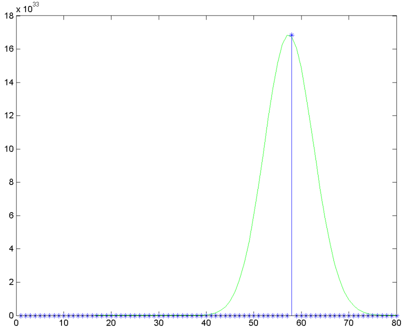
Update of the noise parameters in a closed form: $\alpha^{(\ell+1)}$, $c^{(\ell+1)}$, $\sigma^{(\ell+1)}$ assuming known conditional means $\mathsf{E}_{Q|Y=y,\theta^{(\ell)}}[Q(\mathbf{s})]$ and $\mathsf{E}_{Q|Y=y,\theta^{(\ell)}}[Q(\mathbf{s})^2]$ computed in EM Expectation Step.

EM EXPECTATION STEP

$$\mathsf{E}_{Q|Y=y,\theta^{(\ell)}}[Q(\mathbf{s})] = \frac{\sum_{q_\mathbf{s}=0}^{\infty} \Pi_\mathbf{s}(\theta^{(\ell)}, 1, q_\mathbf{s})}{\sum_{q_\mathbf{s}=0}^{\infty} \Pi_\mathbf{s}(\theta^{(\ell)}, 0, q_\mathbf{s})}$$

Problem: Infinite sums

$$\Pi_\mathbf{s}(\theta, d, q_\mathbf{s}) = \exp\left(-\frac{(y(\mathbf{s}) - \alpha(q_\mathbf{s}+d) - c)^2}{2\sigma^2}\right) \frac{(u_{i_{\mathbb{P}}^{(\ell)}(\mathbf{s})})^{q_\mathbf{s}+d}}{q_\mathbf{s}!}$$



Lemma 3.4.1

Unique maximizer

$$q_\mathbf{s}^* = \frac{\sigma^2}{\alpha^2} \mathsf{W}\left(\frac{\alpha^2}{\sigma^2} u_{i_{\mathbb{P}}^{(\ell)}(\mathbf{s})} e^{\frac{\alpha}{\sigma^2}(y(\mathbf{s})-c-d\alpha)}\right)$$

Proposition 3.4.2

Bounding function: $\mathcal{N}(q_\mathbf{s}^*, \frac{\sigma}{\alpha})$

Bounds:

$q_\mathbf{s}^+ = q_\mathbf{s}^* + \Delta \frac{\sigma}{\alpha}$

$q_\mathbf{s}^- = q_\mathbf{s}^* - \Delta \frac{\sigma}{\alpha}$

EM Expectation step

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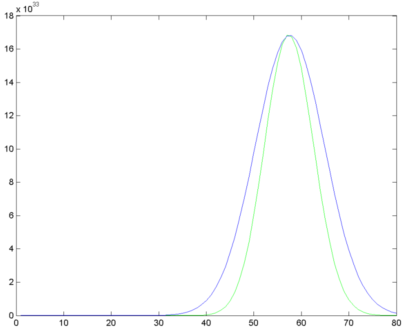

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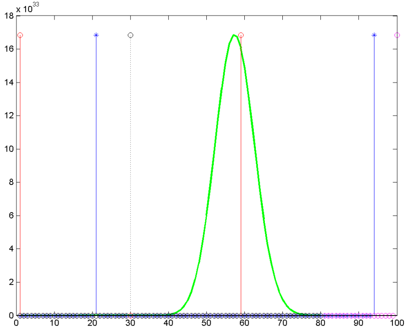

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DISCRETE STEP: GRAPH-CUT

Minimization criterion:

$$i_{\mathbb{P}}^{(\ell)} \in \underset{i_{\mathbb{P}} \in \mathbb{I}}{\operatorname{argmin}} \;\; \Phi(\theta^{(\ell)}, i_{\mathbb{P}}, y) + \rho(i_{\mathbb{P}})$$

$i_{\mathbb{P}}$ takes discrete values
discrete optimization problem

Graph-cut methods:

- Minimizing the energy of a discrete Markov Random Field
- Computationally efficient
- Good convergence properties
  - Global solution for convex ρ
  - Approximation of global minimum for wide class of non-convex ρ

Anisotropic TV Prior

$$\rho(i_{\mathbb{P}}) = \mu\Big(\sum_{n=1}^{N-1} \sum_{m=1}^{M} \psi(|i_{\mathbb{P}}(n+1,m) - i_{\mathbb{P}}(n,m)|) + \sum_{n=1}^{N} \sum_{m=1}^{M-1} \psi(|i_{\mathbb{P}}(n,m+1) - i_{\mathbb{P}}(n,m)|)\Big), \quad \mu \geq 0$$

ANISOTROPIC TV PRIOR

$$\rho(i_{\mathbb{P}}) = \mu\Big(\sum_{n=1}^{N-1}\sum_{m=1}^{M}\psi(|i_{\mathbb{P}}(n+1,m) - i_{\mathbb{P}}(n,m)|)$$
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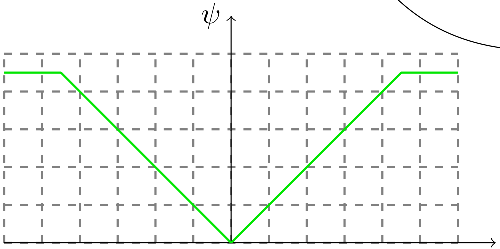
Submodular



[Boykov *et al.* 2001]
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Some interesting cases:

- truncated ℓ_1 norm

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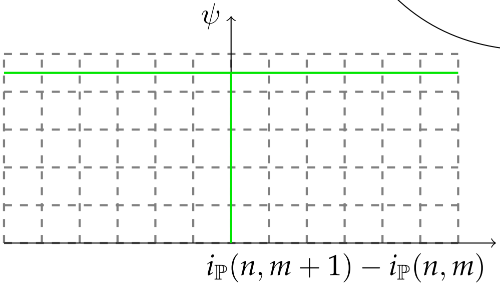

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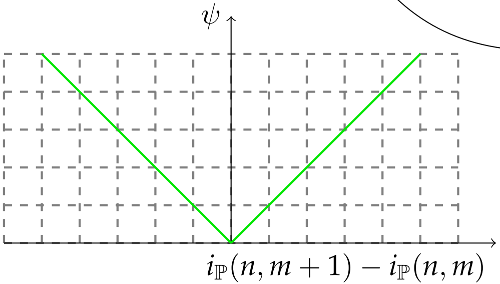
Some interesting cases:

- truncated ℓ_1 norm
- Potts model

Anisotropic TV Prior

$$\rho(i_{\mathbb{P}}) = \mu\Big(\sum_{n=1}^{N-1}\sum_{m=1}^{M}\psi(|i_{\mathbb{P}}(n+1,m) - i_{\mathbb{P}}(n,m)|)$$
$$+\sum_{n=1}^{N}\sum_{m=1}^{M-1}\psi(|i_{\mathbb{P}}(n,m+1) - i_{\mathbb{P}}(n,m)|)\Big), \quad \mu \geq 0$$



Convex

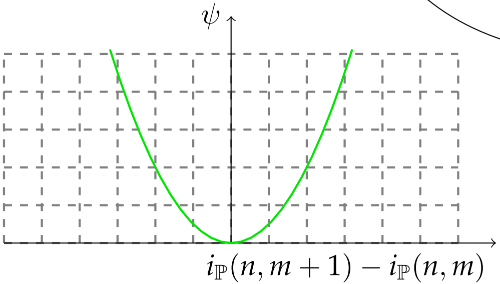
[Ishikawa 2003]
[Murota 2004]
[Kolmogorov *et al.* 2009]

Some interesting cases:

- ℓ_1 norm

Anisotropic TV Prior

$$\rho(i_{\mathbb{P}}) = \mu\Big(\sum_{n=1}^{N-1}\sum_{m=1}^{M}\psi(|i_{\mathbb{P}}(n+1,m) - i_{\mathbb{P}}(n,m)|)$$
$$+ \sum_{n=1}^{N}\sum_{m=1}^{M-1}\psi(|i_{\mathbb{P}}(n,m+1) - i_{\mathbb{P}}(n,m)|)\Big), \quad \mu \geq 0$$



Convex

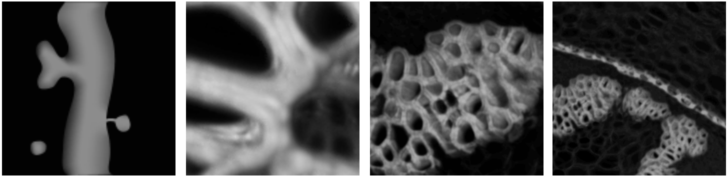
[Ishikawa 2003]
[Murota 2004]
[Kolmogorov *et al.* 2009]

Some interesting cases:

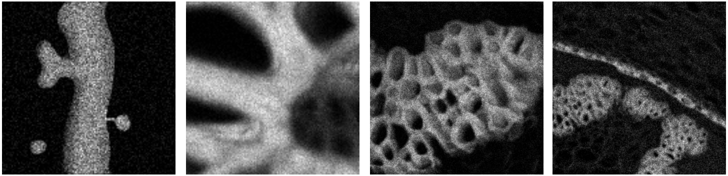
- ℓ_1 norm
- ℓ_2 norm

RESULTS: SINGLE IMAGE NOISE IDENTIFICATION

Original

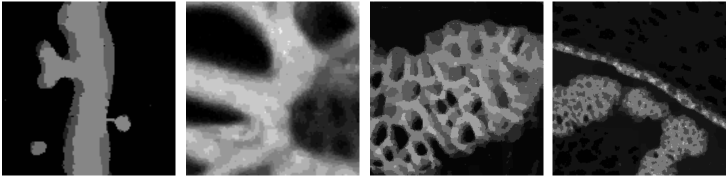


Noisy



$\alpha = 20, c = 100, \sigma^2 = 1000$ | $\alpha = 5, c = 150, \sigma^2 = 125$ | $\alpha = 20, c = 150, \sigma^2 = 1000$ | $\alpha = 30, c = 150, \sigma^2 = 3000$

Results



$\widehat{\alpha} = 19.8, \widehat{c} = 103, \widehat{\sigma}^2 = 1039$ | $\widehat{\alpha} = 4.89, \widehat{c} = 156.35, \widehat{\sigma}^2 = 115.2$ | $\widehat{\alpha} = 19.5, \widehat{c} = 142, \widehat{\sigma}^2 = 907$ | $\widehat{\alpha} = 30.96, \widehat{c} = 149.95, \widehat{\sigma}^2 = 3183$

Summary

Numerical results

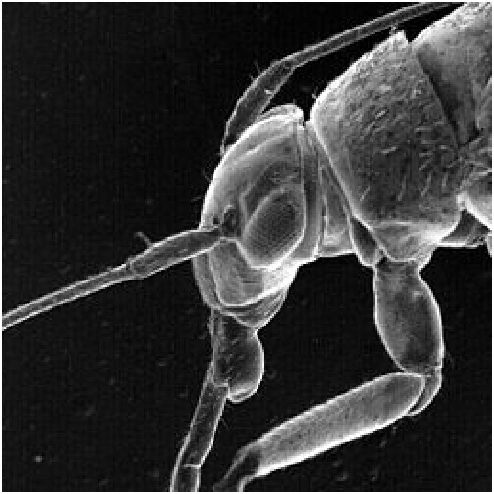
Experiment	σ^2	$\hat{\sigma}^2$	c	$\hat{c}$	α	$\hat{\alpha}$
1	1000	1039	100	103	20	19.8
2	125	115.2	150	156.35	5	4.89
3	1000	907	150	142	20	19.5
4	3000	3183	150	149.95	30	30.96

Table: Estimated noise parameters

RELATED WORK: VECTOR QUANTIZATION

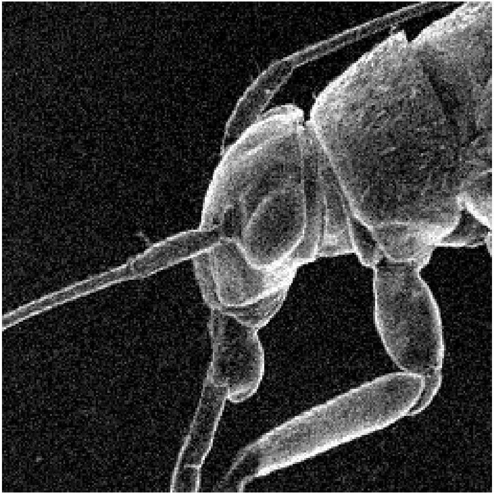
- *Noise parameters are assumed to be known*
- *Only* ***centroids*** *need to be computed* $\Rightarrow$ *Continuous step becomes* ***convex***
- *Continuous step* $\Rightarrow$ *solver:* ***PPXA+ algorithm***
- *Discrete step* $\Rightarrow$ ***unchanged*** *with respect to noise estimation algorithm*
- *Convenient framework to enforce a* ***tunable spatial regularity*** *of variational form*
- *Quantization method based on a* ***two-step*** *procedure intertwining a convex optimization algorithm for quantization level selection and a combinatorial regularization procedure*

Quantization in the presence of noise



256×256 image

Quantization in the presence of noise



256×256 image

$\mathcal{N}(0, 419.5)$
SNR = **11.4** dB

Quantization in the presence of noise

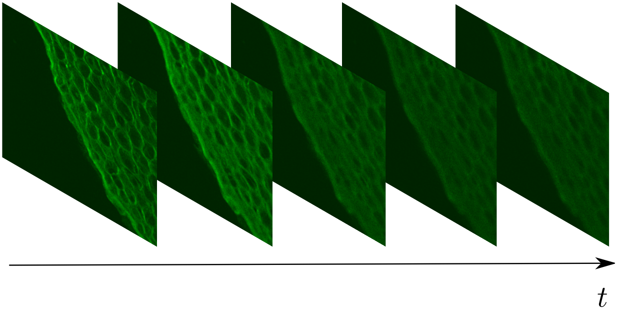
256×256 image

$\mathcal{N}(0, 419.5)$
SNR = **11.4** dB

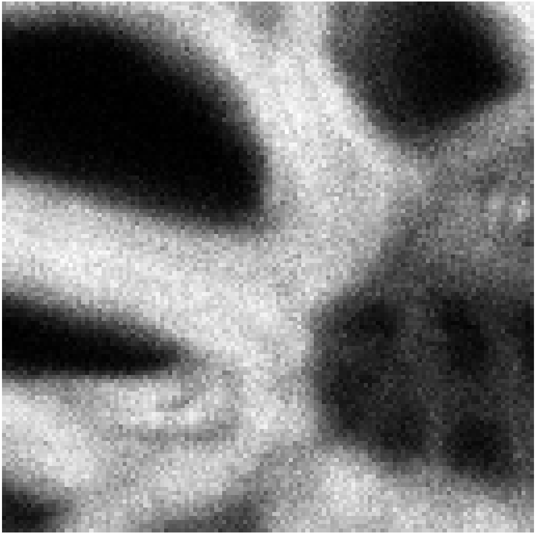
16 quantization levels
Φ - squared ℓ_2 norm
ρ - anisotropic TV
ψ - ℓ_1 norm
$\mu = 500$
SNR = **16.3** dB

Related work: Multiple image noise identification

- *Segmentation step is not required*
- *Signal is nonstationary in time due to the bleaching process*
- *EM algorithm is proposed with Maximization step which additionally includes estimation of the exponential decay rates (Halley algorithm)*
- *Results evaluated in terms of Cramer-Rao bounds*

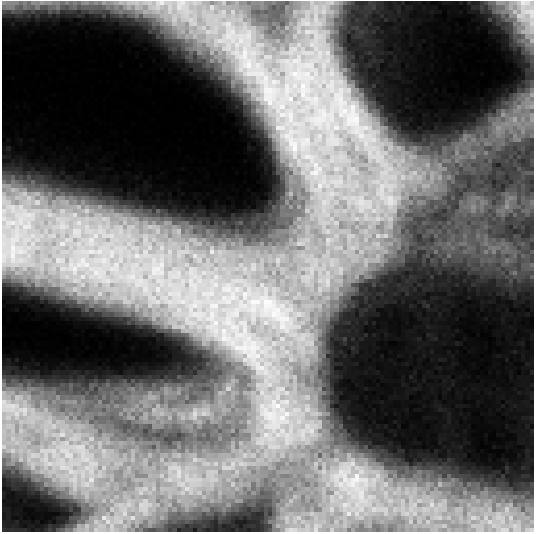


RESULTS: MULTIPLE IMAGE NOISE IDENTIFICATION



Original image: first frame

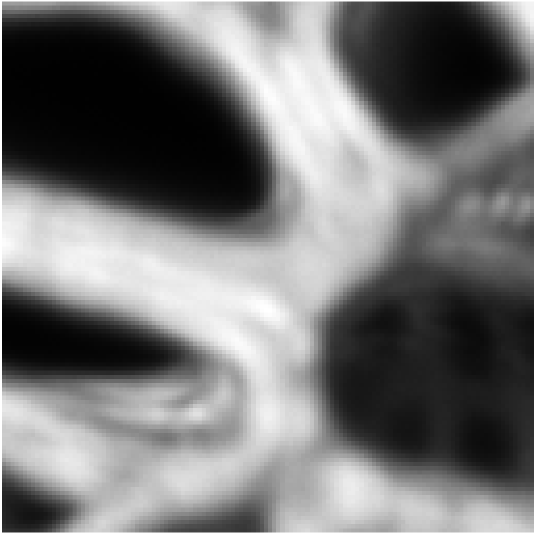
Results: Multiple image noise identification



Original image: first frame

Original image: 180-th frame

Results: Multiple image noise identification

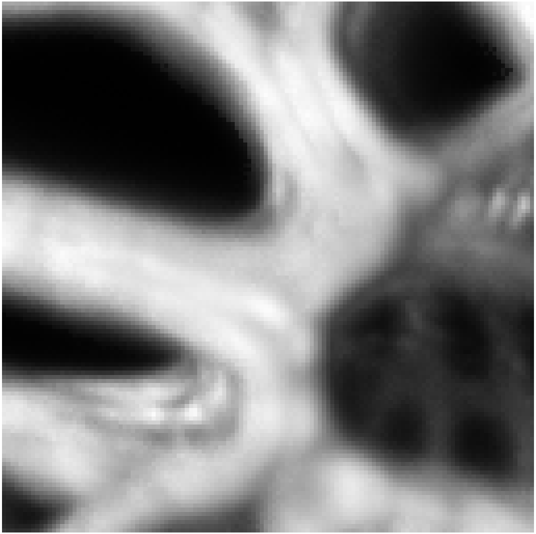


Original image: first frame

Original image: 180-th frame

Mean over $T = 180$ realizations

RESULTS: MULTIPLE IMAGE NOISE IDENTIFICATION



Original image: first frame

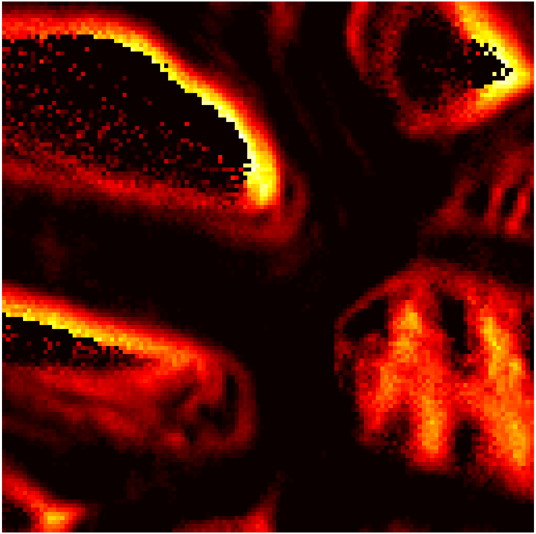
Original image: 180-th frame

Mean over $T = 180$ realizations

Reconstructed image
Parameters:
$\widehat{\alpha} = 25.8, \widehat{c} = 8, \widehat{\sigma}^2 = 119$

$\widehat{u}_s \in [0, 147]$

RESULTS: MULTIPLE IMAGE NOISE IDENTIFICATION



Original image: first frame

Original image: 180-th frame

Mean over $T = 180$ realizations

Reconstructed image

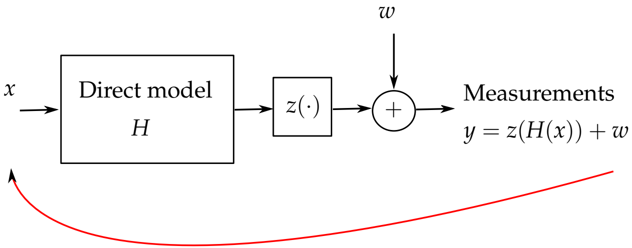
Parameters:
$\widehat{\alpha} = 25.8, \widehat{c} = 8, \widehat{\sigma}^2 = 119$

$\widehat{u}_s \in [0, 147]$

Bleaching map:
$\widehat{k}_s \in [0, 3.9 \times 10^{-6}]$

Image restoration

DEGRADATION MODEL



Which strategy for restoring image x corrupted by Poisson-Gaussian noise?

Method grounded on approximations of the noise statistics

Method based on the true Poisson-Gaussian neg-log likelihood

POISSON-GAUSSIAN DATA FIDELITY TERM

$$\Phi(x) = -\log(p_Y(y;x)) = \sum_{i=1}^{Q} \Phi_i([Hx]_i)$$

Where

$\Phi_i([Hx]_i)$ is given by

$$-\log\left(\sum_{n=0}^{+\infty} \frac{e^{-[Hx]_i}([Hx]_i)^n}{n!} \frac{e^{-\left(\frac{y_i-c-n}{\sqrt{2}\sigma}\right)^2}}{\sqrt{2\pi\sigma^2}}\right)$$

Important questions:

- Is Φ convex?
- What is the explicit form of the proximity operator of Φ?
- Is Φ μ-Lipschitz differentiable?

CONVEXITY

Theorem 5.2.2

The neg-log likelihood $\Phi^{(\beta)}$ of a mixture of Generalized-Gaussian and Poisson variables defined over the positive orthant as

$$\Phi^{(\beta)}(x) = \sum_{i=1}^{Q} \Phi_i^{(\beta)}([Hx]_i)$$

where for all, $i \in \{1, \ldots, Q\}$

$$\Phi_i^{(\beta)}([Hx]_i) = -\log \left(\sum_{n=0}^{+\infty} \frac{e^{-[Hx]_i}([Hx]_i)^n}{n!} \frac{\beta}{2\sqrt{2}\sigma\Gamma(\frac{1}{\beta})} e^{-\left(\frac{|y_i - c - n|}{\sqrt{2}\sigma}\right)^{\beta}} \right)$$

is strictly convex if $\beta > 1$ and convex if $\beta = 1$.

As a special case we have the convexity of Poisson-Gaussian neg-log likelihood

Lipschitz differentiability

Theorem 5.2.1

The function Φ is μ-Lipschitz differentiable on $[0,+\infty)^N$ with

$$\mu = \|H\|^2 \left(1 - e^{-\frac{1}{\sigma^2}}\right) \exp\left(\left(2 \max_{i\in\{1,\dots,Q\}} \{y_i\} - 2c - 1\right) / \sigma^2\right)$$

Gradient:

$$\nabla\Phi(x) = H^\top(\mathbf{1} - \xi(Hx))$$

Hessian:

$$\nabla^2\Phi(x) = H^\top \mathrm{diag}\left(\eta_i([Hx]_i)\right) H$$

$$\forall z = (z_i)_{1\le i\le Q} \in [0,+\infty)^Q,$$
$$\xi(z) = \left(\xi_i(z_i)\right)_{1\le i\le Q},$$
$$\eta(z) = \left(\eta_i(z_i)\right)_{1\le i\le Q}$$

$$\xi_i(z_i) = s(z_i, y_i - c - 1)/s(z_i, y_i - c)$$

$$\eta_i(z_i) = \frac{(s(z_i,y_i-c-1))^2 - s(z_i,y_i-c)s(z_i,y_i-c-2)}{(s(z_i,y_i-c))^2}$$

$$\forall (a,b) \in \mathbb{R}^2,$$
$$s(a,b) = \sum_{n=0}^{+\infty} \frac{a^n}{n!} e^{-\left(\frac{b-n}{\sqrt{2}\sigma}\right)^2}$$

Problem: Infinite sums

Problem formulation

Find

$$\hat{x} \in \underset{x}{\operatorname{argmin}} \quad f(x)$$

Where

$$f(x) = h(x) + \psi_0(x) + \sum_{r=1}^{R} \psi_r(V_r x)$$

- $\psi_r(V_r x)$ - convex regularization term with linear operator $V_r \in \mathbb{R}^{P_r \times N}$
- $\psi_0 \in \Gamma_0(\mathbb{R}^N)$ - indicator function of a closed convex subset of $[0, +\infty)^N$
- $h(x)$ - for non-negative values defined as $-\log(p_{\boldsymbol{Y}}(y \mid x))$ and defined as a quadratic function on $(-\infty, 0]^N$

PROXIMAL METHODS

Proximity operator

The proximity operator of a function $f \in \Gamma_0(\mathcal{X})$ (lower semi-continuous proper convex function) at x is defined as:

$$\forall x \in \mathcal{X}, \quad \operatorname{prox}_f(x) := \underset{p \in \mathcal{X}}{\operatorname{argmin}}\, f(p) + \tfrac{1}{2}\|x - p\|^2$$

♣

Proximal methods incorporating functions either via their proximity operator or via their gradient:

- Forward-backward algorithm ($R = 2$) [Chen and Rockafellar, 1997]
- Forward-backward-forward algorithm ($R = 2$)[Tseng, 2000]
- Generalized forward-backward algorithm ($R \geq 2$) [Raguet *et al.* 2013]
- Primal-dual algorithm ($R \geq 2$) [Vu, 2011] [Condat, 2013] [Combettes and Pesquet, 2012]

Primal-dual splitting approach [Combettes and Pesquet, 2012]

Initialization: $x_0 \in \mathbb{R}^N$, and $(\forall r \in \{1, \ldots, R\})\ v_{r,0} \in \mathbb{R}^{P_r}$
for $k = 0, \ldots$ **do**
$y_{1,k} = x_k - \gamma \left(\nabla h(x_k) + \sum_{r=1}^{R} V_r^\top v_{r,k} \right) + a_k$
$p_{1,k} = \operatorname{prox}_{\gamma \psi_0}(y_{1,k})$
for $r = 1, \ldots, R$ **do**
$y_{2,r,k} = v_{r,k} + \gamma V_r x_k$
$p_{2,r,k} = y_{2,r,k} - \gamma \operatorname{prox}_{\gamma^{-1} \psi_r}(\gamma^{-1} y_{2,r,k})$
$q_{2,r,k} = p_{2,r,k} + \gamma V_r p_{1,k}$
$v_{r,k+1} = v_{r,k} - y_{2,r,k} + q_{2,r,k}$
end for
$q_{1,k} = p_{1,k} - \gamma \left(\nabla h(p_{1,k}) + \sum_{r=1}^{R} V_r^\top p_{2,r,k} \right) + c_k$
$x_{k+1} = x_k - y_{1,k} + q_{1,k}$
end for

Primal-dual splitting approach [Combettes and Pesquet, 2012]

Initialization: $x_0 \in \mathbb{R}^N$, and $(\forall r \in \{1, \ldots, R\})$ $v_{r,0} \in \mathbb{R}^{P_r}$
for $k = 0, \ldots$ **do**

$y_{1,k} = x_k - \gamma \left(\nabla h(x_k) + \sum_{r=1}^{R} V_r^\top v_{r,k} \right) + a_k$

$p_{1,k} = \mathrm{prox}_{\gamma \psi_0}(y_{1,k})$

for $r = 1, \ldots, R$ **do**

$y_{2,r,k} = v_{r,k} + \gamma V_r x_k$

$p_{2,r,k} = y_{2,r,k} - \gamma\, \mathrm{prox}_{\gamma^{-1}\psi_r}(\gamma^{-1} y_{2,r,k})$

$q_{2,r,k} = p_{2,r,k} + \gamma V_r p_{1,k}$

$v_{r,k+1} = v_{r,k} - y_{2,r,k} + q_{2,r,k}$

end for

$q_{1,k} = p_{1,k} - \gamma \left(\nabla h(p_{1,k}) + \sum_{r=1}^{R} V_r^\top p_{2,r,k} \right) + c_k$

$x_{k+1} = x_k - y_{1,k} + q_{1,k}$

end for

Primal-dual splitting approach [Combettes and Pesquet, 2012]

Initialization: $x_0 \in \mathbb{R}^N$, and $(\forall r \in \{1,\dots,R\})\ v_{r,0} \in \mathbb{R}^{P_r}$
for $k = 0, \dots$ **do**
$y_{1,k} = x_k - \gamma \left(\nabla h(x_k) + \sum_{r=1}^{R} V_r^\top v_{r,k} \right) + a_k$
$p_{1,k} = \mathrm{prox}_{\gamma\psi_0}(y_{1,k})$
for $r = 1, \dots, R$ **do**
$y_{2,r,k} = v_{r,k} + \gamma V_r x_k$
$p_{2,r,k} = y_{2,r,k} - \gamma\, \mathrm{prox}_{\gamma^{-1}\psi_r}(\gamma^{-1} y_{2,r,k})$
$q_{2,r,k} = p_{2,r,k} + \gamma V_r p_{1,k}$
$v_{r,k+1} = v_{r,k} - y_{2,r,k} + q_{2,r,k}$
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$q_{1,k} = p_{1,k} - \gamma \left(\nabla h(p_{1,k}) + \sum_{r=1}^{R} V_r^\top p_{2,r,k} \right) + c_k$
$x_{k+1} = x_k - y_{1,k} + q_{1,k}$
end for

CONVERGENCE

Assumptions:

❶ f is coercive, i.e. $\lim_{\|x\|\to+\infty} f(x) = +\infty$,

❷ for every $r \in \{1, \ldots, R\}$, ψ_r is finite valued,

❸ $\gamma \in [\epsilon, (1-\epsilon)/\delta]$ where $\epsilon \in (0, 1/(\delta+1))$ and $\delta = \mu + \sqrt{\sum_{r=1}^{R} \|V_r\|^2}$,

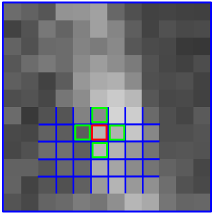
❹ $(a_k)_{k\in\mathbb{N}}$ and $(c_k)_{k\in\mathbb{N}}$ are absolutely summable sequences.

Result

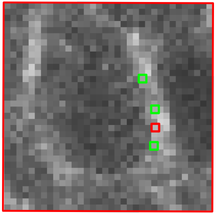
There exists a minimizer $\overline{x}$ of $f(x)$ such that the sequences $(x_k)_{k\in\mathbb{N}}$ and $(p_{1,k})_{k\in\mathbb{N}}$ converge to $\overline{x}$.

♣

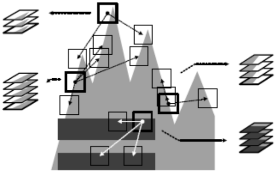
Wide range of penalization strategies



TV



NLTV



BM3D

TV
[Rudin *et al.* 1992]

NLTV
[Gilboa and Osher *et al.* 2008]

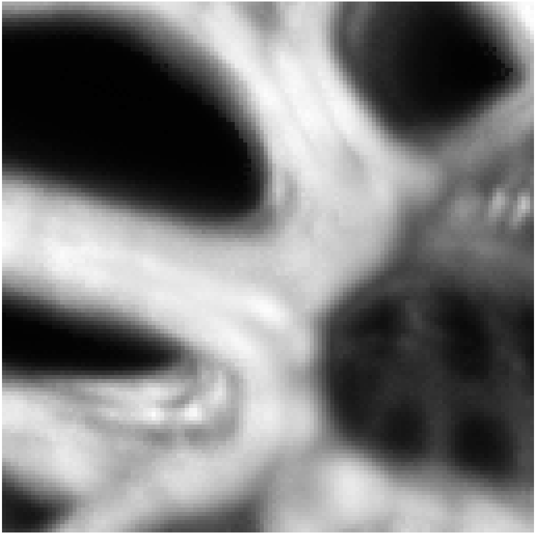
Hessian-TV
[Lefkimmiatis *et al.* 2012]

BM3D frames
[Danielyan *et al.* 2012]

...

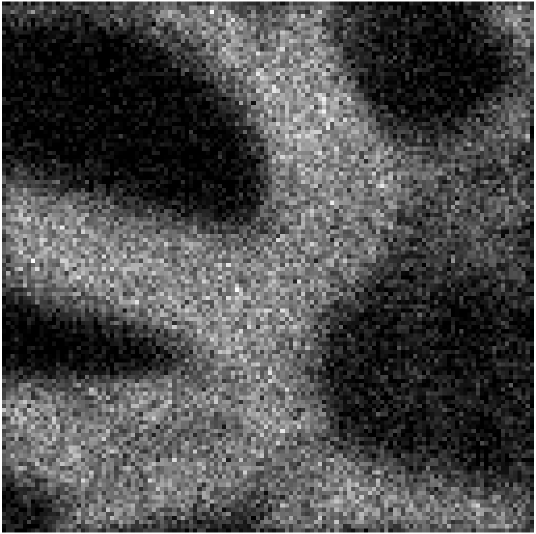
Flexibility: Large range of penalization strategies can be applied

RESULTS



Original image:
size 128×128

RESULTS



Original image:
size 128×128

Noisy blurred image:
25×25 truncated Gaussian blur with std 1.6
$\sigma^2 = 12$
MAE = 35
SNR = 10.07

Results

Original image:
size 128×128

Noisy blurred image:
25×25 truncated Gaussian blur with std 1.6
$\sigma^2 = 12$
MAE = 35
SNR = 10.07

Reconstructed image (Hessian-TV prior):
MAE = 7.91
SNR = 21.52

Results

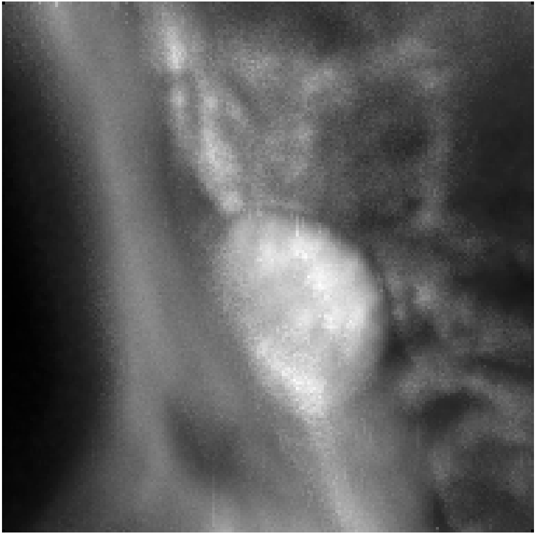
Original image:
size 128×128

Noisy blurred image:
25×25 truncated Gaussian blur with std 1.6
$\sigma^2 = 12$
MAE = 35
SNR = 10.07

Reconstructed image (Hessian-TV prior):
MAE = 7.91
SNR = 21.52

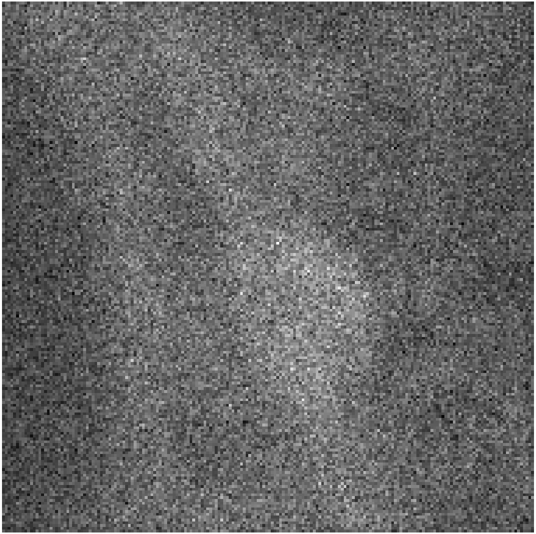
(BM3D prior):
ongoing work
MAE = 7.99
SNR = 21.86

RESULTS



Original image:
size 190×190

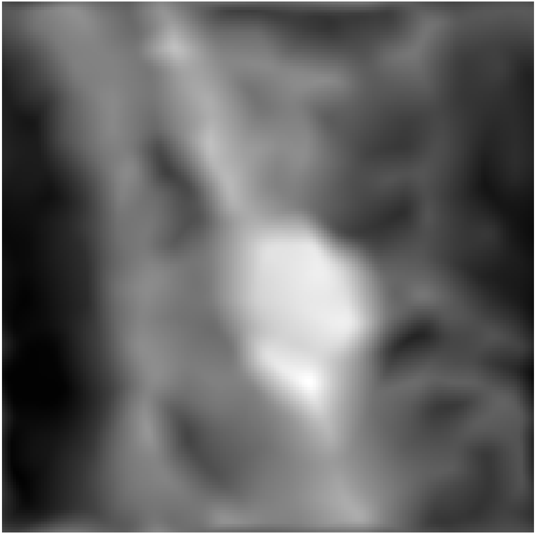
Results



Original image:
size 190×190

Noisy blurred image:
25×25 truncated Gaussian blur with std 1.6
$\sigma^2 = 9$
MAE = 61
SNR = 2.19

RESULTS



Original image:
size 190×190

Noisy blurred image:
25×25 truncated Gaussian blur with std 1.6
$\sigma^2 = 9$
MAE = 61
SNR = 2.19

Reconstructed image (Hessian-TV prior):
MAE = 7.79
SNR = 19.53

$\ell_2 - \ell_0$ SPARSITY MEASURES

Regularization term: $\rho(x) = \sum_{r=1}^{R} \psi_r(V_r(x))$

Common assumption: x has a sparse representation with respect to V_r

Question: What sparsity measure ψ_r should be used?

Theoretically: ℓ_0 measure [Donoho *et al.* 1995]

Usually: ℓ_1 measure

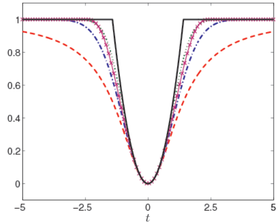
We examine $\ell_2 - \ell_0$ sparsity measures

Discrete approach

2 submodular moves which alternate

Convex move [Murota 2000]

Quantized move [Jezierska *et al.* 2011]



Continuous approach

Majorize-Minimize alg.

Extension of [Chouzenoux *et al.* 2011]

ℓ_0 penalty obtained asymptotically

New convergence proof

RESULTS: $\ell_2 - \ell_0$ SPARSITY MEASURES

Original image:
size 256×256

RESULTS: $\ell_2 - \ell_0$ SPARSITY MEASURES

Original image:
size 256×256

Noisy blurred image:
3×3 uniform blur Noise:
$\mathcal{N}(0, 16)$
SNR = 18.65
MSSIM = 0.82

Results: $\ell_2 - \ell_0$ sparsity measures

Original image:
size 256×256

Noisy blurred image:
3×3 uniform blur Noise: $\mathcal{N}(0, 16)$
SNR = 18.65
MSSIM = 0.82

Reconstructed image Hessian-TV prior (3MG SC):
SNR = 26.90
MSSIM = 0.94

RESULTS: $\ell_2 - \ell_0$ SPARSITY MEASURES

Original image:
size 256×256

Noisy blurred image:
3×3 uniform blur Noise:
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Reconstructed image Hessian-TV prior (3MG SC):
SNR = 26.90
MSSIM = 0.94

(3MG SNC2):
(TV: Geman-McClure sparsity measure)
SNR = 27.96
MSSIM = 0.94

RESULTS: $\ell_2 - \ell_0$ SPARSITY MEASURES - DETAILS

Original image:
size 256×256

Noisy blurred image:
3×3 uniform blur Noise: $\mathcal{N}(0, 16)$
SNR = 18.65
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Reconstructed image Hessian-TV prior (3MG SC):
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RESULTS: $\ell_2 - \ell_0$ SPARSITY MEASURES - DETAILS

Original image:
size 256×256

Noisy blurred image:
3×3 uniform blur Noise:
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Contributions: Algorithms

- An optimization framework featuring both continuous and combinatorial techniques.
- Properties of Poisson-Gaussian neg log likelihood.
- Approximation for the Poisson-Gaussian neg log likelihood.
- Numerical method for a maximum likelihood estimate of Poisson-Gaussian distribution hyperparameters and its Cramer-Rao bounds.
- Image denoising problem with $\ell_2 - \ell_0$ sparsity measures, in discrete and continuous frameworks.

Contributions: Algorithms

- An optimization framework featuring both continuous and combinatorial techniques.
- Properties of Poisson-Gaussian neg log likelihood.
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- Noise identification techniques useful for calibration systems and restoration algorithms (from time series data and from single image).
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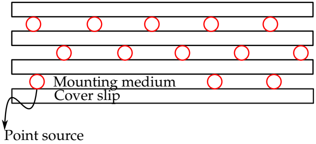
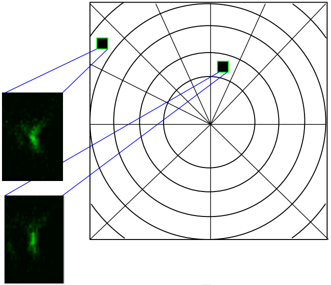
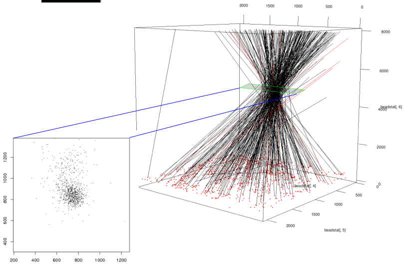
ONGOING WORK

Confocal macroscopy optical PSF estimation

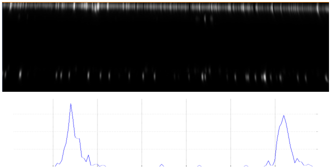
Experiment:







Intensity profile:



Perspectives

- Study of BM3D prior for Poisson-Gaussian data restoration
- Restoration of Generalized Poisson-Gaussian data
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- Comparison with other Primal-Dual algorithms
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Thank you

CONTINUOUS STEP: EM ALGORITHM

EM algorithm

$$(\forall \ell \in \mathbb{N}) \qquad \theta^{(\ell+1)} = \underset{\theta \in \mathcal{T}}{\operatorname{argmin}} \; \Phi(\theta \mid \theta^{(\ell)})$$

Expectation step

$$\Phi(\theta \mid \theta^{(\ell)}) = \mathsf{E}_{Q|Y=y,\theta^{(\ell)}}[\ln p_{Y,Q}(Y, Q \mid \theta)] \qquad Q \text{ - missing data}$$

$$\Phi(\theta \mid \theta^{(\ell)}) = \frac{1}{2\sigma^2} \sum_{\mathbf{s} \in \mathbb{S}} \mathsf{E}_{Q|Y=y,\theta^{(\ell)}}[(y(\mathbf{s}) - \alpha Q(\mathbf{s}) - c)^2] + \frac{NM}{2} \ln(\sigma^2)$$
$$+ \sum_{k=1}^{K} \operatorname{card}(\mathbb{D}_k^{(\ell)}) u_k - \sum_{k=1}^{K} \ln u_k \sum_{\mathbf{s} \in \mathbb{D}_k^{(\ell)}} \mathsf{E}_{Q|R=r,\theta^{(\ell)}}[Q(\mathbf{s})]$$

Maximization step

$$\theta^{(\ell+1)} = \underset{\theta \in \mathcal{T}}{\operatorname{argmin}} \; \Phi(\theta \mid \theta^{(\ell)})$$

(x^+, σ^2)		Init.	Poiss.	Gauss.	GAST	Exact
(15,9)	λ	-	0.145	0.139	0.069	0.079
	MAE	54.26	13.29	10.86	11.38	**10.60**
	SNR	6.31	18.68	19.74	19.27	**19.89**
	SSIM	0.088	0.659	0.730	0.736	**0.747**
(30,12)	λ	-	0.105	0.120	0.056	0.048
	MAE	34.81	9.60	8.46	8.71	**8.25**
	SNR	10.72	21.13	21.60	21.41	**21.85**
	SSIM	0.179	0.752	0.811	0.807	**0.812**
(60,30)	λ	-	0.076	0.069	0.032	0.032
	MAE	26.72	8.28	7.41	7.44	**7.28**
	SNR	12.34	22.23	22.67	22.67	**22.85**
	SSIM	0.255	0.783	**0.845**	0.834	0.839
(90,50)	λ	-	0.065	0.052	0.022	0.025
	MAE	22.67	7.43	6.64	6.59	**6.55**
	SNR	13.73	23.13	23.59	23.79	**23.82**
	SSIM	0.312	0.804	**0.864**	0.855	0.859
(120,60)	λ	-	0.047	0.042	0.017	0.018
	MAE	19.64	6.71	6.11	**5.92**	**5.92**
	SNR	14.85	24.01	24.39	24.62	**24.67**
	SSIM	0.367	0.829	0.876	**0.877**	**0.877**
(150,80)	λ	-	0.046	0.032	0.016	0.016
	MAE	18.17	6.61	5.94	**5.85**	**5.85**
	SNR	15.46	24.02	24.59	24.54	**24.65**
	SSIM	0.399	0.829	**0.878**	**0.878**	**0.878**

		Poiss.	Gauss.	GAST	Exact
TV	λ	0.163	0.197	0.093	0.083
	MAE	10.71	9.42	9.70	**8.90**
	SNR	20.21	20.6	20.55	**21.22**
	SSIM	0.715	0.777	0.782	**0.802**
NLTV	λ	0.105	0.120	0.056	0.048
	MAE	9.60	8.46	8.71	**8.25**
	SNR	21.13	21.60	21.41	**21.85**
	SSIM	0.752	0.811	0.807	**0.812**
TV + Hessian	λ_{TV}	0.042	0.258	0.026	0.032
	λ_{H}	0.148	0.376	0.070	0.082
	MAE	8.99	7.92	8.10	**7.91**
	SNR	21.09	**21.52**	21.40	**21.52**
	SSIM	0.794	**0.854**	0.851	**0.854**

		Poiss.	Gauss.	GAST	Exact
TV	λ	0.394	0.254	0.176	0.158
	MAE	11.58	9.02	10.16	**8.66**
	SNR	16.7	18.49	17.49	**18.81**
	SSIM	0.643	0.670	0.660	**0.679**
NLTV	λ	0.283	0.197	0.138	0.138
	MAE	11.80	9.33	10.35	**9.27**
	SNR	16.69	18.28	17.37	**18.29**
	SSIM	0.622	0.643	0.632	**0.644**
TV + Hessian	λ_{TV}	0.079	0.167	0.125	0.119
	λ_{H}	0.856	0.690	0.582	0.346
	MAE	10.69	7.84	9.13	**7.79**
	SNR	17.32	19.48	18.38	**19.53**
	SSIM	0.726	**0.755**	0.742	**0.755**