A Convex Variational Approach for Restoring Data Corrupted with Poisson-Gaussian Noise

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CONTENTS

POISSON-GAUSSIAN NOISE

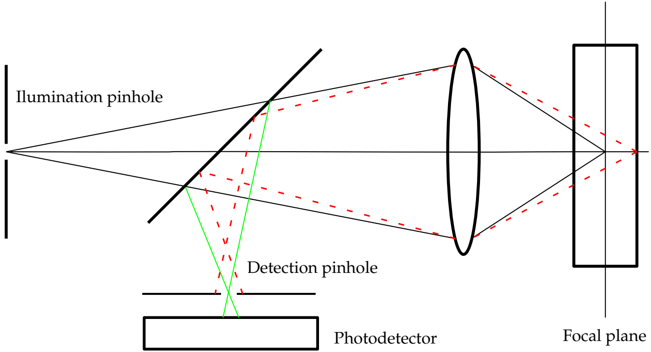
Physical acquisition process:

- *Various noise sources*
  - *Some are signal dependent, e.g. photon noise*
  - *Some are signal independent, e.g. electronic noise, thermal noise etc.*
- *Too complex for either Poisson or additive Gaussian noise models*
- *Next simplest model: sum of Poisson and Gaussian models for signal dependent and signal independent components, respectively.*
- *A recent topic of interest in the literature [Snyder* et al. *1993] [Delpretti* et al. *2008] [Benvenuto* et al. *2008] [Luisier* et al. *2011] [Gil-Rodrigo* et al. *2011] [Chakrabarti and Zickler 2012] [Li* et al. *2012]*

Imaging related application areas:

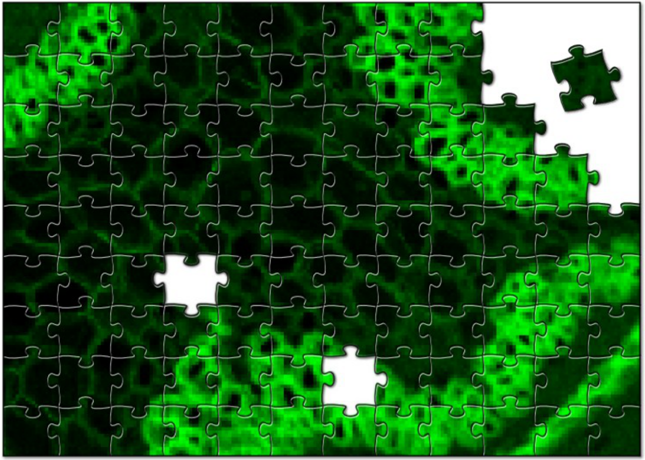
- *Biology (Fluorescence imaging systems)*
- *Astronomy*
- *CCD camera imaging*

OUR MAIN MOTIVATION: CONFOCAL IMAGING



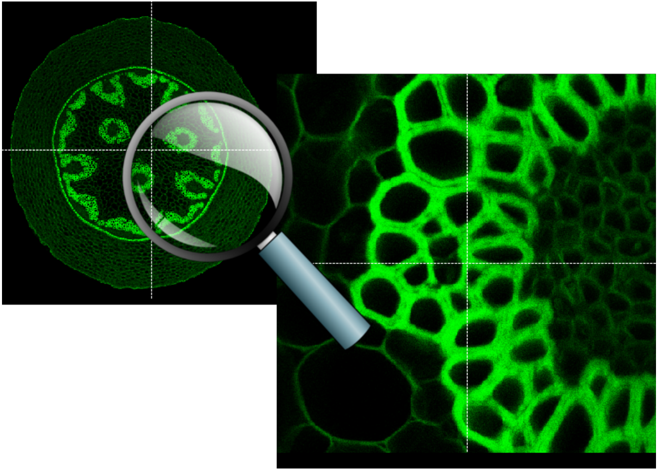
- *Much of the out-of-focus light eliminated from detection*
- *High optical resolution (sample depth direction)*
- *Image acquired point-by-point (small number of detectable photons: down to 8-12 photons per highest intensity pixel)*
- *Application area: biomedical research*

SEEING THE WOOD FOR THE TREES



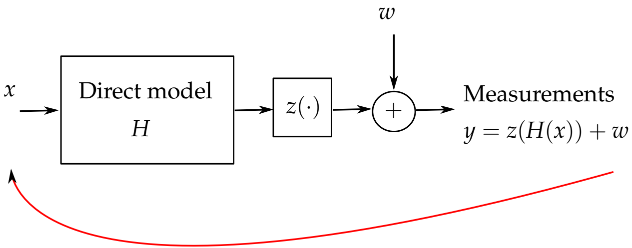
Confocal microscopy

SEEING THE WOOD FOR THE TREES



Confocal macroscopy

DEGRADATION MODEL



Which strategy for restoring image x corrupted by Poisson-Gaussian noise?

Method grounded on approximations of the noise statistics

Method based on the true Poisson-Gaussian neg-log likelihood

DEGRADATION MODEL

$$y = \alpha \; z(x) + w$$

Observations

$y \in \mathcal{Y}$
$y = (y_i)_{i \in \mathbb{Y}}$

[Zhang '07] [Boulanger *et al.* '08] [Delpretti *et al.* '08] [Luisier *et al.* '11]

DEGRADATION MODEL

Poisson noise

$x \in \mathcal{X}$ - original signal

$x = (x_i)_{i \in \mathbb{X}}$ $\quad x_i \in [0, +\infty)$ $Z_i(x) \sim \mathcal{P}([Hx]_i))$

$H : \mathcal{X} \mapsto \mathcal{Y}$ - blur

$z(x) = \big(z_i(x)\big)_{i \in \mathbb{X}}$ - realization of $\big(Z_i(x)\big)_{i \in \mathbb{X}}$

$\alpha \in (0, +\infty)$ - scale parameter

$$y = \alpha \; z(x) + w$$

Observations

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$$y = \alpha\, z(x) + w$$

Observations

$y \in \mathcal{Y}$

$y = (y_i)_{i \in \mathbb{Y}}$

White Gaussian noise

$W_i \sim \mathcal{N}(c, \sigma^2)$

$w = (w_i)_{i \in \mathbb{X}}$ - realization of $\big(W_i\big)_{i \in \mathbb{X}}$

$c \in \mathbb{R}$ - mean σ^2 - variance

[Zhang '07] [Boulanger *et al.* '08] [Delpretti *et al.* '08] [Luisier *et al.* '11]

POISSON-GAUSSIAN DATA FIDELITY TERM

$$\Phi(x) = -\log(p_Y(y;x)) = \sum_{i=1}^{Q} \Phi_i([Hx]_i)$$

Where

$\Phi_i([Hx]_i)$ is given by

$$-\log\left(\sum_{n=0}^{+\infty} \frac{e^{-[Hx]_i}([Hx]_i)^n}{n!} \frac{e^{-\left(\frac{y_i-c-n}{\sqrt{2}\sigma}\right)^2}}{\sqrt{2\pi\sigma^2}}\right)$$

Important questions:

- Is Φ convex?
- What is the explicit form of the proximity operator of Φ?
- Is Φ μ-Lipschitz differentiable?

CONVEXITY

Theorem 5.2.2

The neg-log likelihood $\Phi^{(\beta)}$ of a mixture of Generalized-Gaussian and Poisson variables defined over the positive orthant as

$$\Phi^{(\beta)}(x) = \sum_{i=1}^{Q} \Phi_i^{(\beta)}([Hx]_i)$$

where for all, $i \in \{1, \ldots, Q\}$

$$\Phi_i^{(\beta)}([Hx]_i) = -\log\left(\sum_{n=0}^{+\infty} \frac{e^{-[Hx]_i}([Hx]_i)^n}{n!} \frac{\beta}{2\sqrt{2}\sigma\Gamma(\frac{1}{\beta})} e^{-\left(\frac{|y_i-c-n|}{\sqrt{2}\sigma}\right)^{\beta}}\right)$$

is strictly convex if $\beta > 1$ and convex if $\beta = 1$.

As a special case we have the convexity of Poisson-Gaussian neg-log likelihood

Lipschitz differentiability

Theorem 5.2.1

The function Φ is μ-Lipschitz differentiable on $[0,+\infty)^N$ with

$$\mu = \|H\|^2 \left(1 - e^{-\frac{1}{\sigma^2}}\right) \exp\left(\left(2 \max_{i \in \{1,\dots,Q\}} \{y_i\} - 2c - 1\right)/\sigma^2\right)$$

Gradient:

$\nabla\Phi(x) = H^\top(\mathbf{1} - \xi(Hx))$

Hessian:

$\nabla^2\Phi(x) =$

$H^\top \mathrm{diag}\left(\eta_i([Hx]_i)\right) H$

$\forall z = (z_i)_{1\le i\le Q} \in [0,+\infty)^Q$,

$\xi(z) = \left(\xi_i(z_i)\right)_{1\le i\le Q}$,

$\eta(z) = \left(\eta_i(z_i)\right)_{1\le i\le Q}$

$\xi_i(z_i) = s(z_i, y_i - c - 1)/s(z_i, y_i - c)$

$\eta_i(z_i) = \frac{(s(z_i,y_i-c-1))^2 - s(z_i,y_i-c)s(z_i,y_i-c-2)}{(s(z_i,y_i-c))^2}$

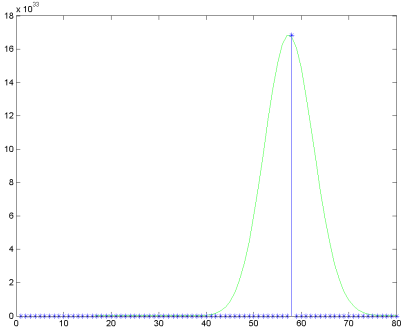
$\forall (a,b) \in \mathbb{R}^2$,

$s(a,b) = \sum_{n=0}^{+\infty} \frac{a^n}{n!} e^{-\left(\frac{b-n}{\sqrt{2}\sigma}\right)^2}$

Problem: Infinite sums

INFINITE SUMS PROBLEM [JEZIERSKA ET AL., 2013]

$$s(a,b) = \sum_{n=0}^{+\infty} \pi(a,b,n) \quad \text{where} \quad \Pi(a,b,n) = \frac{a^n}{n!} e^{-\left(\frac{b-n}{\sqrt{2}\sigma}\right)^2}$$



Lemma 3.4.1

Unique maximizer

$$n^* = \sigma^2 \mathsf{W}\left(\frac{a}{\sigma^2} e^{b/\sigma^2}\right)$$

Proposition 3.4.2

Bounding function: $\mathcal{N}(q_{\mathbf{s}}^*, \frac{\sigma}{\alpha})$

Bounds:

$n^- = \lfloor n^* - \Delta\sigma \rfloor$

$n^+ = \lceil n^* + \Delta\sigma \rceil$

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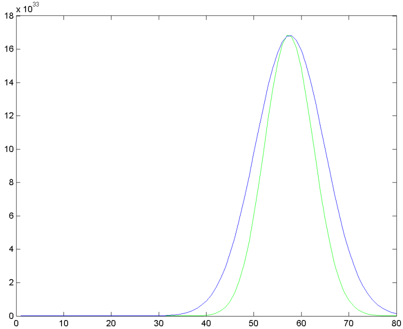

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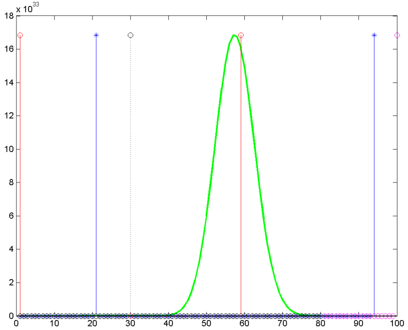

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PROBLEM FORMULATION

Find

$$\hat{x} \in \underset{x}{\operatorname{argmin}} \quad f(x)$$

Where

$$f(x) = h(x) + \psi_0(x) + \sum_{r=1}^{R} \psi_r(V_r x)$$

- $\psi_r(V_r x)$ - convex regularization term with linear operator $V_r \in \mathbb{R}^{P_r \times N}$
- $\psi_0 \in \Gamma_0(\mathbb{R}^N)$ - indicator function of a closed convex subset of $[0, +\infty)^N$
- $h(x)$ - for non-negative values defined as $-\log(p_{\boldsymbol{Y}}(y \mid x))$ and defined as a quadratic function on $(-\infty, 0]^N$

Proximal methods

Proximity operator

The proximity operator of a function $f \in \Gamma_0(\mathcal{X})$ (lower semi-continuous proper convex function) at x is defined as:

$$\forall x \in \mathcal{X}, \quad \mathrm{prox}_f(x) := \underset{p \in \mathcal{X}}{\operatorname{argmin}}\, f(p) + \frac{1}{2}\|x - p\|^2$$

Proximal methods incorporating functions either via their proximity operator or via their gradient:

- Forward-backward algorithm ($R = 2$) [Chen and Rockafellar, 1997]
- Forward-backward-forward algorithm ($R = 2$)[Tseng, 2000]
- Generalized forward-backward algorithm ($R \geq 2$) [Raguet *et al.* 2013]
- Primal-dual algorithm ($R \geq 2$) [Vu, 2011] [Condat, 2013] [Combettes and Pesquet, 2012]

Primal-dual splitting approach [Combettes and Pesquet, 2012]

Initialization: $x_0 \in \mathbb{R}^N$, and $(\forall r \in \{1,\ldots,R\})$ $v_{r,0} \in \mathbb{R}^{P_r}$
for $k = 0, \ldots$ **do**

$$y_{1,k} = x_k - \gamma \left(\nabla h(x_k) + \sum_{r=1}^{R} V_r^\top v_{r,k} \right) + a_k$$
$$p_{1,k} = \mathrm{prox}_{\gamma\psi_0}(y_{1,k})$$

for $r = 1, \ldots, R$ **do**

$$y_{2,r,k} = v_{r,k} + \gamma V_r x_k$$
$$p_{2,r,k} = y_{2,r,k} - \gamma\, \mathrm{prox}_{\gamma^{-1}\psi_r}(\gamma^{-1} y_{2,r,k})$$
$$q_{2,r,k} = p_{2,r,k} + \gamma V_r p_{1,k}$$
$$v_{r,k+1} = v_{r,k} - y_{2,r,k} + q_{2,r,k}$$

end for

$$q_{1,k} = p_{1,k} - \gamma \left(\nabla h(p_{1,k}) + \sum_{r=1}^{R} V_r^\top p_{2,r,k} \right) + c_k$$
$$x_{k+1} = x_k - y_{1,k} + q_{1,k}$$

end for

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$\quad q_{1,k} = p_{1,k} - \gamma\left(\nabla h(p_{1,k}) + \sum_{r=1}^{R} V_r^\top p_{2,r,k}\right) + c_k$
$\quad x_{k+1} = x_k - y_{1,k} + q_{1,k}$
end for

Convergence

Assumptions:

1. f is coercive, i.e. $\lim_{\|x\|\to+\infty} f(x) = +\infty$,
2. for every $r \in \{1,\dots,R\}$, ψ_r is finite valued,
3. $\gamma \in [\epsilon, (1-\epsilon)/\delta]$ where $\epsilon \in (0, 1/(\delta+1))$ and $\delta = \mu + \sqrt{\sum_{r=1}^{R} \|V_r\|^2}$,
4. $(a_k)_{k\in\mathbb{N}}$ and $(c_k)_{k\in\mathbb{N}}$ are absolutely summable sequences.

Result

There exists a minimizer $\overline{x}$ of $f(x)$ such that the sequences $(x_k)_{k\in\mathbb{N}}$ and $(p_{1,k})_{k\in\mathbb{N}}$ converge to $\overline{x}$.

♣

DATA FIDELITY TERMS

- Exact Poisson-Gaussian model

$$-\log(p_Y(y;x)) = \sum_{i=1}^{Q} -\log\left(\sum_{n=0}^{+\infty} \frac{e^{-[Hx]_i}([Hx]_i)^n}{n!} \frac{e^{-\left(\frac{y_i-n}{\sqrt{2}\sigma}\right)^2}}{\sqrt{2\pi\sigma^2}}\right)$$

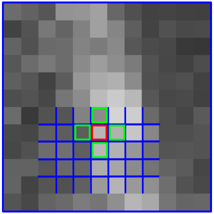
Compared with

- Poisson model
- Gaussian model
- Generalized Anscombe transform (GAST) model [Murtagh *et al.* 1995]

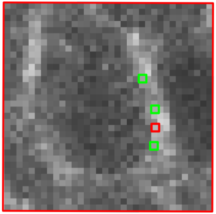
$$-\log(p_Y(y;x)) = \sum_{i=1}^{Q} (\nu([Hx]_i) - \nu(y))^2, \quad \nu(\theta) = \sqrt{\theta + \frac{3}{8} + \sigma^2}$$

Flexibility: Large range of data fidelity terms can be applied

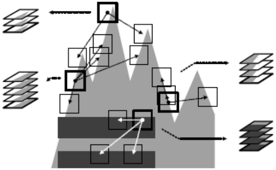
WIDE RANGE OF PENALIZATION STRATEGIES



TV



NLTV



BM3D

TV
[Rudin *et al.* 1992]

NLTV
[Gilboa and Osher *et al.* 2008]

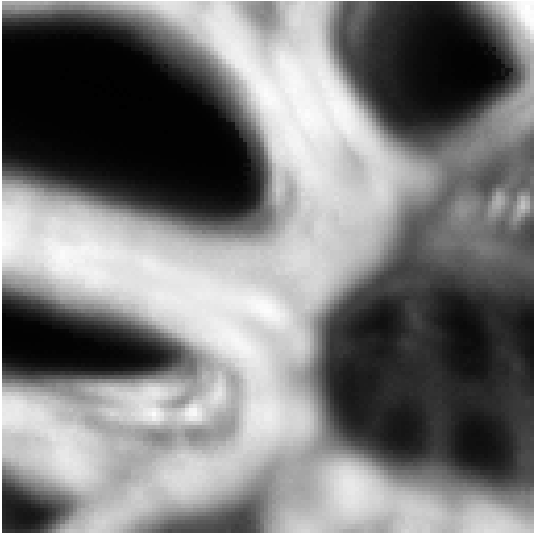
Hessian-TV
[Lefkimmiatis *et al.* 2012]

BM3D frames
[Danielyan *et al.* 2012]

...

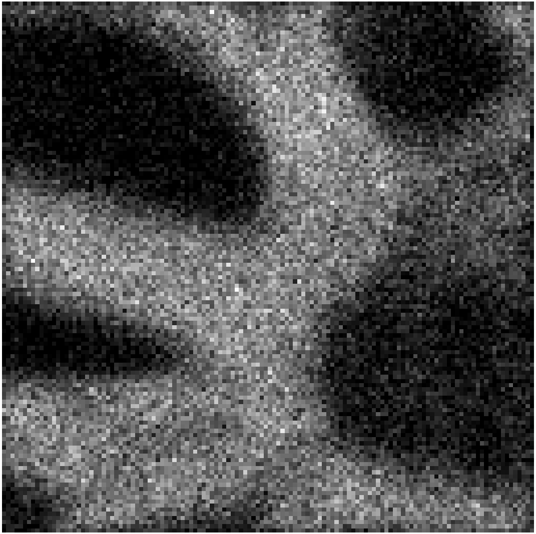
Flexibility: Large range of penalization strategies can be applied

RESULTS



Original image:
size 128×128

Results



Original image:
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Noisy blurred image:
25×25 truncated Gaussian blur with std 1.6
$\sigma^2 = 12$
MAE = 35
SNR = 10.07

Results

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Reconstructed image (Hessian-TV prior):
MAE = 7.91
SNR = 21.52

RESULTS

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MAE = 35
SNR = 10.07

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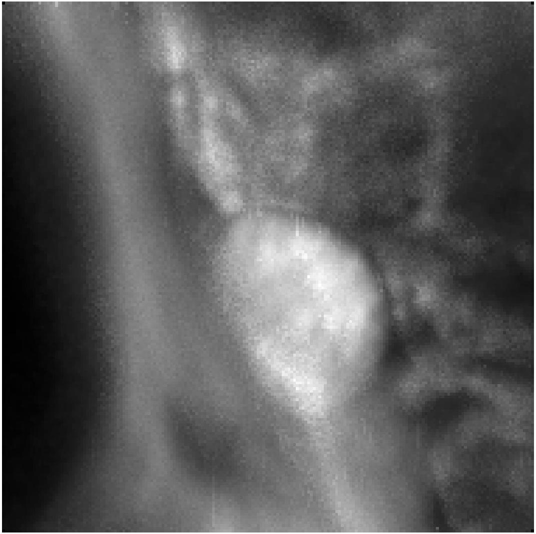
MAE = 7.91
SNR = 21.52

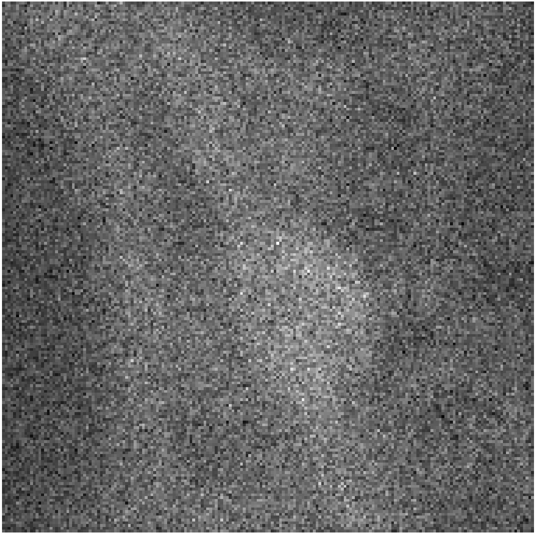
(BM3D prior):
ongoing work
MAE = 7.99
SNR = 21.86

Results



Original image:
size 190×190

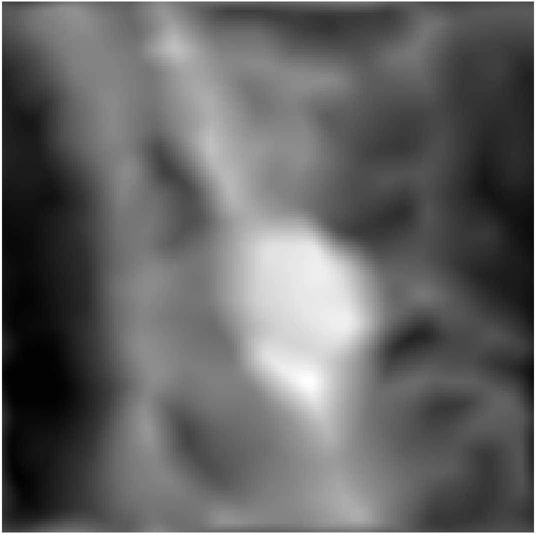
Results



Original image:
size 190×190

Noisy blurred image:
25×25 truncated Gaussian blur with std 1.6
$\sigma^2 = 9$
MAE = 61
SNR = 2.19

Results



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$\sigma^2 = 9$
MAE = 61
SNR = 2.19

Reconstructed image (Hessian-TV prior):
MAE = 7.79
SNR = 19.53

		Poiss.	Gauss.	GAST	Exact
TV	λ	0.163	0.197	0.093	0.083
	MAE	10.71	9.42	9.70	**8.90**
	SNR	20.21	20.6	20.55	**21.22**
	SSIM	0.715	0.777	0.782	**0.802**
NLTV	λ	0.105	0.120	0.056	0.048
	MAE	9.60	8.46	8.71	**8.25**
	SNR	21.13	21.60	21.41	**21.85**
	SSIM	0.752	0.811	0.807	**0.812**
TV + Hessian	λ_{TV}	0.042	0.258	0.026	0.032
	λ_{H}	0.148	0.376	0.070	0.082
	MAE	8.99	7.92	8.10	**7.91**
	SNR	21.09	**21.52**	21.40	**21.52**
	SSIM	0.794	**0.854**	0.851	**0.854**

		Poiss.	Gauss.	GAST	Exact
TV	λ	0.394	0.254	0.176	0.158
	MAE	11.58	9.02	10.16	**8.66**
	SNR	16.7	18.49	17.49	**18.81**
	SSIM	0.643	0.670	0.660	**0.679**
NLTV	λ	0.283	0.197	0.138	0.138
	MAE	11.80	9.33	10.35	**9.27**
	SNR	16.69	18.28	17.37	**18.29**
	SSIM	0.622	0.643	0.632	**0.644**
TV + Hessian	λ_{TV}	0.079	0.167	0.125	0.119
	λ_{H}	0.856	0.690	0.582	0.346
	MAE	10.69	7.84	9.13	**7.79**
	SNR	17.32	19.48	18.38	**19.53**
	SSIM	0.726	**0.755**	0.742	**0.755**

CONCLUSIONS

- Properties of Poisson-Gaussian neg-log-likelihood
  - $\rightsquigarrow$ Convexity
  - $\rightsquigarrow$ Lipschitz differentiability
- Primal-dual splitting algorithm
  - $\rightsquigarrow$ High flexibility
  - $\rightsquigarrow$ Robust to computational errors
- Future work
  - $\rightsquigarrow$ Variable metric acceleration for primal-dual algorithm
  - $\rightsquigarrow$ Non-convex regularization strategies for Poisson-Gaussian data
  - $\rightsquigarrow$ Comparison with other approximation of Poisson-Gaussian data fidelity term present in the literature

Various approximations od Poisson-Gaussian data fidelity term

Scaled gradient method [Benvenuto *et al.* 2008]

Gradient of neg-log likelihood approximated by:

$$H^\top \left(1 - \exp\left(-\frac{1 + 2([Hx]_i) - y_i}{2([Hx]_i) + \sigma^2}\right)\right)_{i \in \mathbb{X}}$$

Alternating-minimization alg. [Gil-Rodrigo *et al.* 2011]

Neg-log likelihood approximated by:

$$\frac{1}{2}\left\|\left(D_{\widetilde{\sigma}^2}(Hx)\right)^{-1/2}(y - Hx)\right\|_2^2$$

Covariance matrix $D_{\widetilde{\sigma}^2}$ depends on noise parameters

Augmented Lagrangian method [Chakrabarti and Zickler 2012]

Neg-log likelihood approximated by:

$$\sum_{i \in \mathbb{X}} [Hx]_i + \sigma^2 - \left(y_i + \sigma^2\right) \log\left([Hx]_i + \sigma^2\right)$$

Augmented Lagrangian method [Li *et al.* 2012]

Neg-log likelihood approximated by:

$$\frac{1}{2}\left\|\left(\widetilde{\sigma}^2 I + (Hx)\right)^{-1/2}(y - Hx)\right\|_2^2$$

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