POISSON-GAUSSIAN NOISE PARAMETER ESTIMATION IN FLUORESCENCE MICROSCOPY IMAGING

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INTRODUCTION

State of the art:

✗ Non-optimality (unknown parameters not jointly estimated)

Proposed method:

✓ Iterative, flexible algorithm producing reliable results

PROBLEM

Poisson-Gaussian model: $R_{s,t} = \alpha Q_{s,t} + W_{s,t}$

$$Q_{s,t} \sim \mathcal{P}\left(u_s e^{-k_s t}\right) \qquad W_{s,t} \sim \mathcal{N}(c, \sigma^2)$$

$u = (u_s)_{1\le s\le S} \in [0,+\infty)^S$ - "true" signal
$k = (k_s)_{1\le s\le S} \in [0,+\infty)^S$ - bleaching rate
c, σ^2 - mean & variance of Gaussian noise
$\alpha \in \mathbb{R}$ - Poisson noise scaling parameter

Problem formulation:

$$\text{Find } \hat\theta = \left(\hat u, \hat k, \hat\alpha, \hat c, \hat\sigma^2\right)$$

$\theta = (u, k, \alpha, c, \sigma^2)$ - unknown parameters
$r = (r_{s,t})_{1\le s\le S, 1\le t\le T}$ - observed vector
$R = (R_{s,t})_{1\le s\le S, 1\le t\le T}$ - random field
t - time index and s -space index

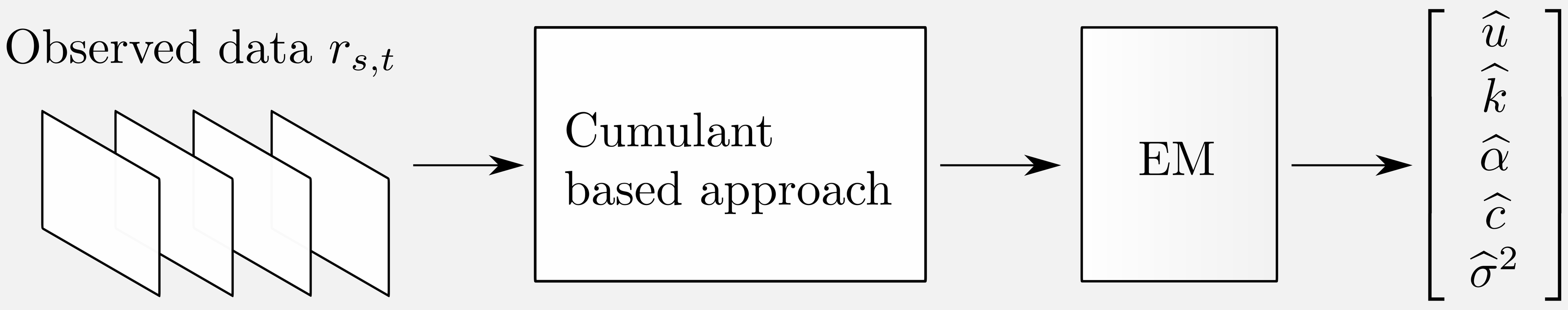
Challenge:

Find $2S+3$ parameters with ST measurements

Assumptions:

u - deterministic
Q and W - mutually independent
Elements of W (resp Q) - independent

APPROACH



CUMULANT BASED INITIALIZATION

Cumulant of order n:

$$\kappa_n[R_{s,t}] = \alpha^n \kappa_n[Q_{s,t}] + \kappa_n[W_{s,t}]$$

mean: $\kappa_1[R_{s,t}] = \alpha e^{-k_s t} u_s + c$
variance: $\kappa_2[R_{s,t}] = \alpha^2 e^{-k_s t} u_s + \sigma^2$

Observation 1: $R_{s,t} = a_s e^{-k_s t} + c + E_{s,t}, \quad (a_s = \alpha u_s)_{1\le s\le S}$
$(E_{s,t})_{1\le s\le S, 1\le t\le T}$ - independent zero-mean random variables

$$(\widehat{a}, \widehat{k}, \widehat{c}) = \underset{a,k,c}{\text{argmin}} \sum_{s=1}^{S}\sum_{t=1}^{T}\left(r_{s,t} - c - a_s e^{-k_s t}\right)^2$$

Alternating minimization approach:

Initialization: $c^{(0)} = \min\{\frac{1}{T}\sum_{t=1}^{T} r_{s,t}, 1 \le s \le S\}$
For $n = 1\ldots N$
  For $s = 1\ldots S$
    $(a_s^{(n)}, k_s^{(n)}) = \underset{a_s, k_s \ge 0}{\text{argmin}} \sum_{t=1}^{T}\left(r_{s,t} - c^{(n-1)} - a_s e^{-k_s t}\right)^2$
  $c^{(n)} = \sum_{s=1}^{S}\sum_{t=1}^{T}(r_{s,t} - a_s^{(n)} e^{-k_s^{(n)} t})/ST$
$\widehat{a} = a^{(N)}, \widehat{k} = k^{(N)}, \widehat{c} = c^{(N)}$

Observation 2: $\mathsf{E}[(R_{s,t} - \mathsf{E}[R_{s,t}])^2] = \alpha a_s e^{-k_s t} + \sigma^2$
$\rightsquigarrow$ Least squares estimator for α
$\rightsquigarrow (\forall s \in \{1,\ldots,S\}) \quad \widehat{u}_s = \frac{\widehat{a}_s}{\widehat{\alpha}}$
$\rightsquigarrow \widehat{\sigma}^2 = \left(\sum_{(s,t)\in\mathbb{I}}\left(v_{s,t} - \widehat{\alpha}\widehat{a}_s e^{-\widehat{k}_s t}\right)\right)/\text{card}(\mathbb{I}),$
where $v_{s,t} = (r_{s,t} - \widehat{a}_s e^{-\widehat{k}_s t} - \widehat{c})^2$, $\mathbb{I}$ - set for imposing positivity

EM

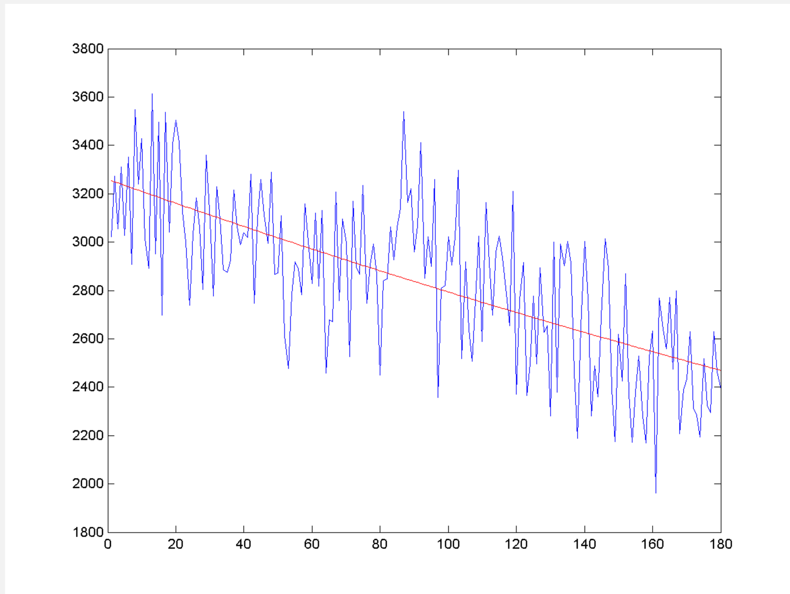
Iterative algorithm: $(\forall n \in \mathbb{N}) \qquad \theta^{(n+1)} = \underset{\theta}{\text{argmax}} \;\; J(\theta \mid \theta^{(n)})$

$$J(\theta \mid \theta^{(n)}) = \mathsf{E}_{Q|R=r,\theta^{(n)}}[\ln p_{R,Q}(R,Q \mid \theta)]$$

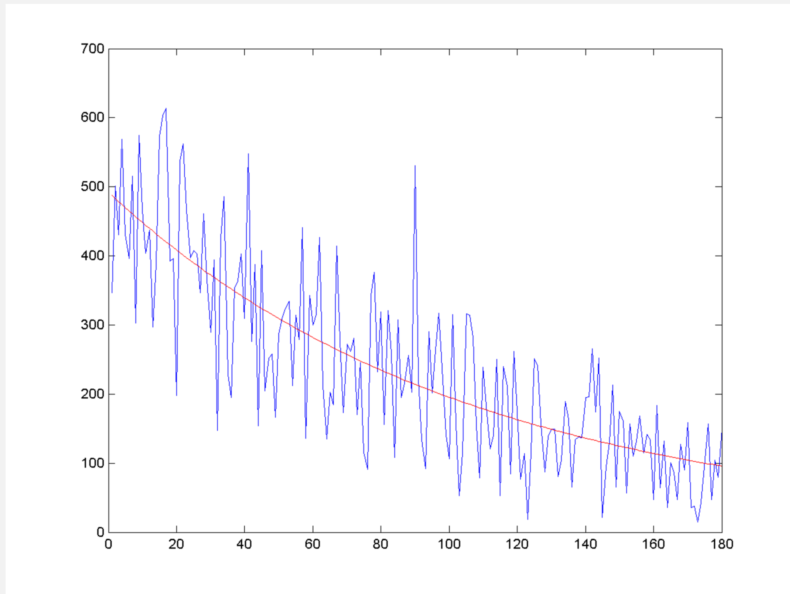
$p_{R,Q}(R,Q \mid \theta)$ - mixed continuous-discrete probability measure

- Main difficulties: computation of $\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}]$ and $\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}^2]$
- EM step based on [1]

TIME VARIATIONS



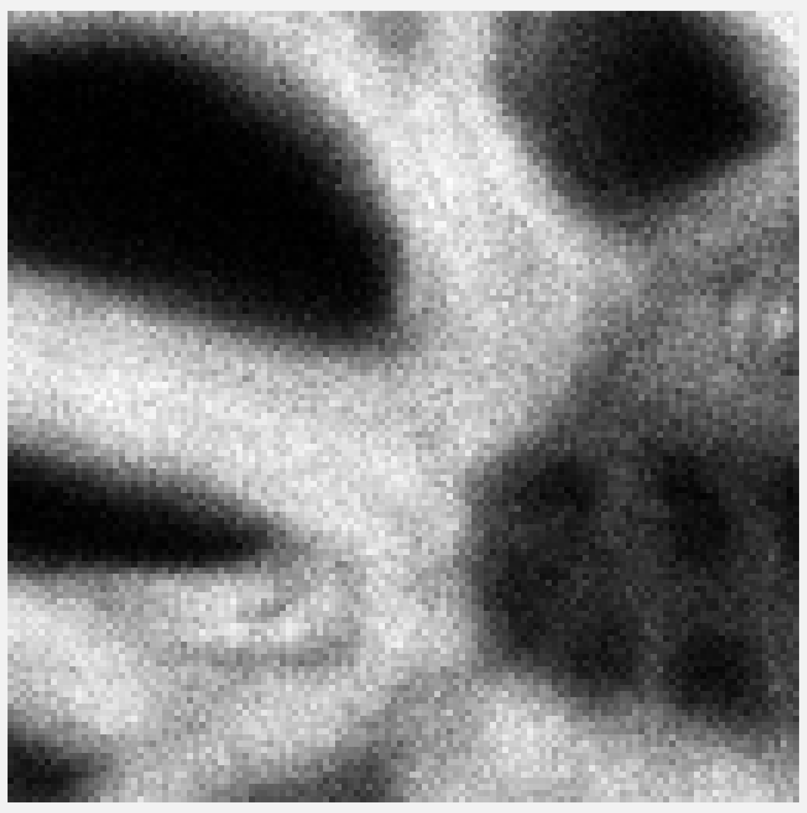
$25.8 \times 126\, e^{-5.7\times10^{-7} t} + 8$



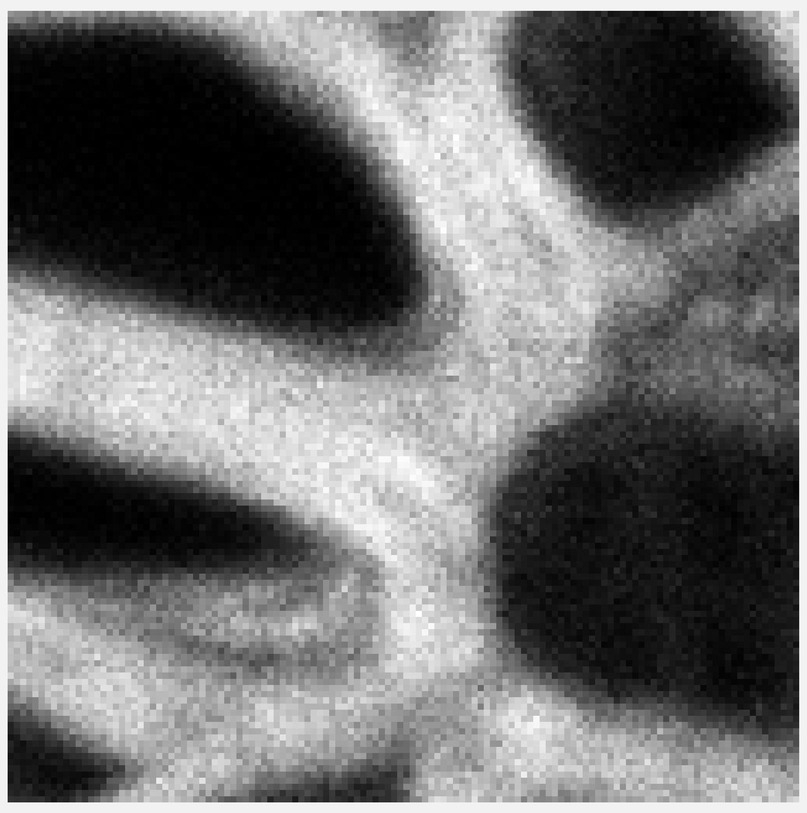
$25.8 \times 19\, e^{-3.6\times10^{-6} t} + 8$

Observed data , *Reconstructed*

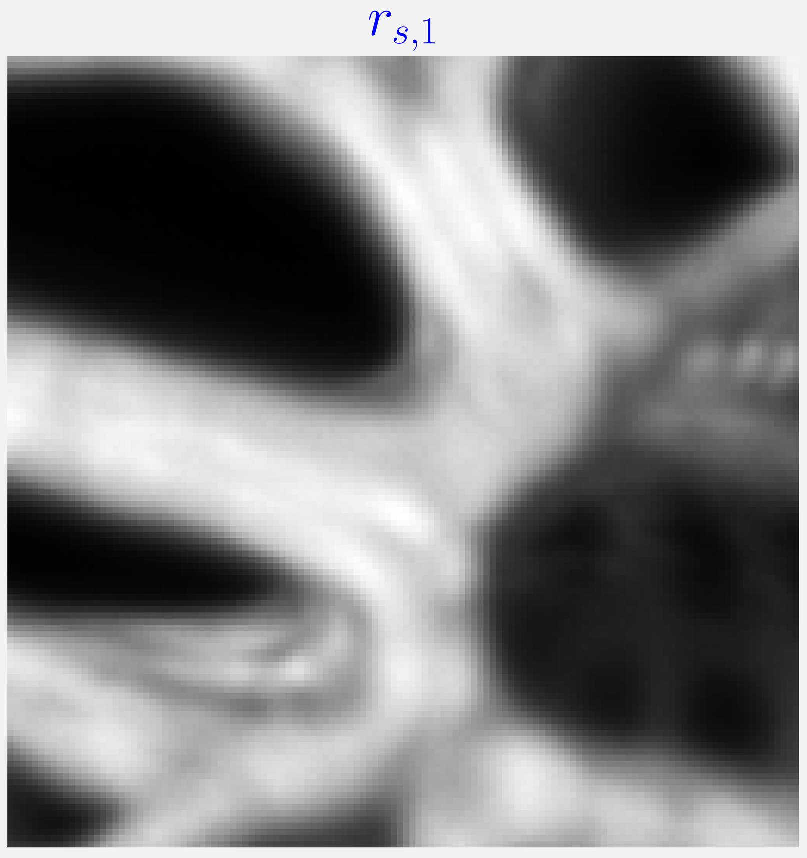
VISUAL RESULTS



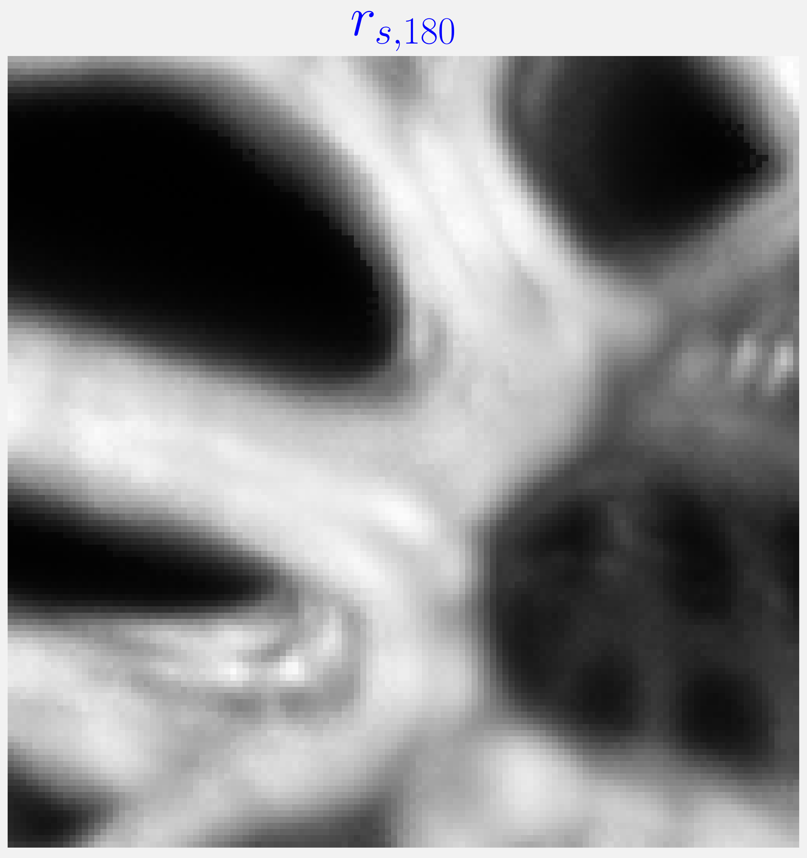
$r_{s,1}$



$r_{s,180}$



$\frac{1}{T}\sum_{t=1}^{T} r_{s,t}$



Reconstructed $\widehat{u}_s$

Images from macro confocal laser scanning microscope (Leica TCS-LSI)

Method	$\hat\sigma$		$\hat c$		$\hat\alpha$		SNR
	bias	std	bias	std	bias	std	
Init.	357.5	3.1	1.9	1.0	-0.3	0.4	39.5
EM	2.9	0.9	1.4	0.8	-0.3	0.4	39.7

Synthetic data results for $c = 10$, $\alpha = 30$, $\sigma^2 = 100$ *and* $T = 180$

References

[1] A. Jezierska, C.Chaux, J.-C. Pesquet, and H. Talbot , *An EM approach for Poisson-Gaussian noise modeling*, European Signal Processing Conference (EUSIPCO), Barcelona, 29 August - 2 September 2011