A fast solver for truncated-convex priors: quantized-convex split moves

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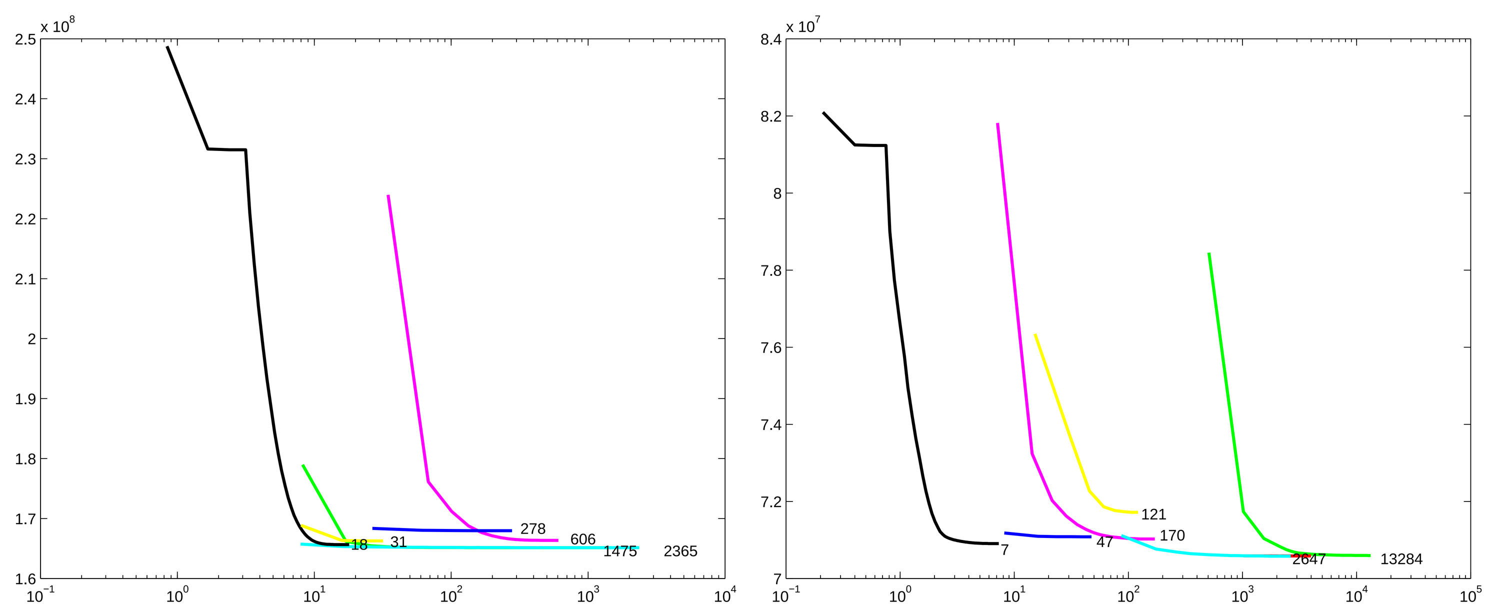
INTRODUCTION

State of the art:

✗ Complexity of Graph Cut based algorithms increases steeply with the number of labels [Veksler 2007],[Kumar & Torr 2008]

Proposed algorithm:

✓ A fast solver for problems with a high number of labels, resulting in a good compromise between efficiency and precision



Truncated ℓ_2 Truncated ℓ_1-ℓ_2

Energy versus log time characteristics: ICM, BP-S, BP, TRW-S, $\alpha\beta$ swap, α-exp, ours.

PROBLEM

Notations:

- $\mathcal{L} = \{0, 1, \dots, L\}$ - ordered discrete set of labels
- L - maximum label in $\mathcal{L}$
- $\mathcal{G}(\mathcal{V}, \mathcal{E})$ - undirected graph with set of edges $\mathcal{E}$ and set of vertices $\mathcal{V}$
- x_u - label assigned to node $u \in \mathcal{V}$
- $\mathcal{N}$ - considered neighborhood
- $c(u, v)$ - cost between $(u, v) \in \mathcal{N}$

Goal of the algorithm:

$$\underset{x}{\text{minimize}} \ \sum_{u\in\mathcal{V}} \mathcal{D}(x_u) + \lambda \sum_{(u,v)\in\mathcal{E}} \mathcal{R}(x_u, x_v)$$

λ - positive real value
$\mathcal{D}$ - some measure of data fidelity
$\mathcal{R}$ - regularization term defined as:

$$\mathcal{R}(x_v, x_u) = \begin{cases} f(x_u - x_v) & \text{if } |x_u - x_v| < T \\ f(T) & \text{if } |x_u - x_v| \geq T \end{cases}$$

Image denoising application:

- Truncated priors: combine noise suppression with edge preservation
- Multilabel problem, e.g. $L = 255$
- $T \approx 30$

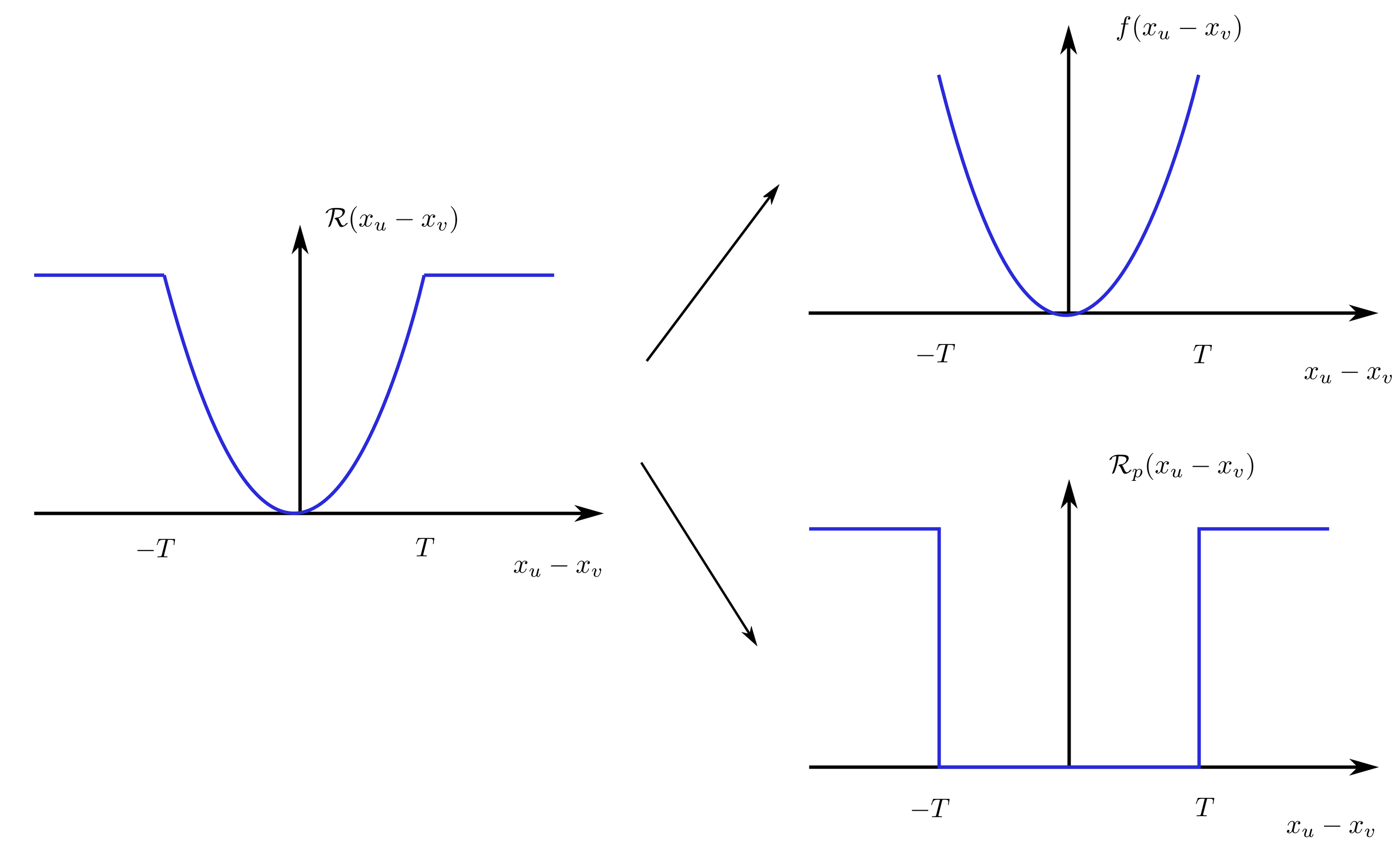
ALGORITHM

❶ Initialize $x^{(0)}$

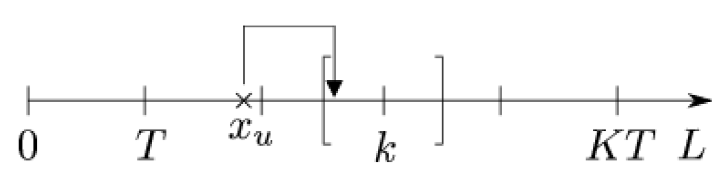
❷ Step 1: Quantized move loop (The result of this step is accepted only if the original Energy is reduced)

❸ Step 2: Convex move loop

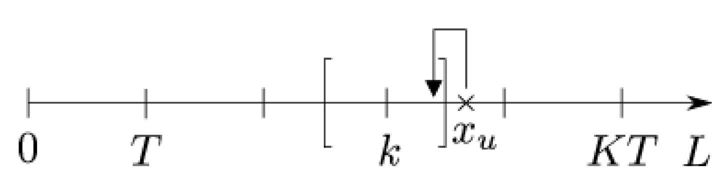
Energy minimization achieved by repeating Steps 1 and Step 2 in the loop.



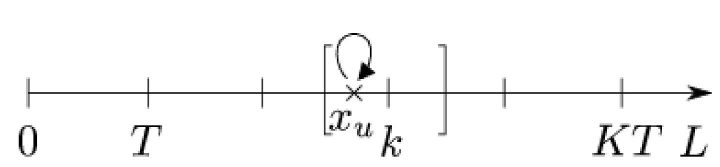
OPTIMIZATION



Case 1: x_u is below $\mathcal{T}_k$



Case 2: x_u is above $\mathcal{T}_k$



Case 3: x_u is inside $\mathcal{T}_k$

$$\alpha(x_u, k) = \begin{cases} t_1^k & \text{if } x_u \leq t_1^k \\ t_T^k & \text{if } x_u \geq t_T^k, \end{cases}$$

Step 1: Quantized move

Submodular, binary, similar to α-expansion move [Boykov et al. 2001], minimizing energy with prior defined as $\mathcal{R}_p(x_v, x_u) =$

$$\begin{cases} 0 & \text{if } |x_u - x_v| < T \\ f(T) & \text{if } |x_u - x_v| \geq T, \end{cases}$$

- $k \in \mathcal{K} = \{k_0, k_1, \dots, k_K\}$
- $k_0 = 0$, $k_i = iT$, $i \in \mathbb{N}_+$
- $KT \geq L$ and $(K-1)T < L$
- $\mathcal{T}^k = \{t_1^k, \dots, t_T^k\}$ st. $t_{i+1}^k = t_i^k + 1$
- e.g. $\mathcal{T}^k = \{k - \frac{T}{2} + 1, \dots, k + \frac{T}{2}\}$
- $x_u^{(n+1)}$ either changes to $\alpha(x_u^{(n)}, k)$ or remains unchanged

Step 2: Convex move

Submodular, binary move, minimizing energy with prior defined as convex function $f(x_u - x_v)$ [Murota 2004], [Kolmogorov et al. 2009].

- $\mathcal{S}$ - set of discrete values s from $\mathbb{Z}$, here $\mathcal{S} = \{-1, 1\}$
- $x_u^{(n+1)}$ either changes to $x_u^{(n)} + s$ or remains unchanged
- $c(u, v) = f(|x_u + s - x_v|) + f(|x_u - x_v - s|) - 2f(|x_u - x_v|)$ if $|x_u - x_v| < T$ and 0 otherwise (similar concept to [Rother et al. 2005])

Original

Noisy G(0,25.3)

TRW-S 20.16 dB

Ours 20.35 dB