Image quantization under spatial smoothness constraints

Anna Jezierska, Caroline Chaux, Hugues Talbot and Jean-Christophe Pesquet

Université Paris-Est, Institut Gaspard Monge, UMR-CNRS 8049
F-77454 Marne la Vallée, Cedex 2, France
e-mail: {first.last}@univ-paris-est.fr

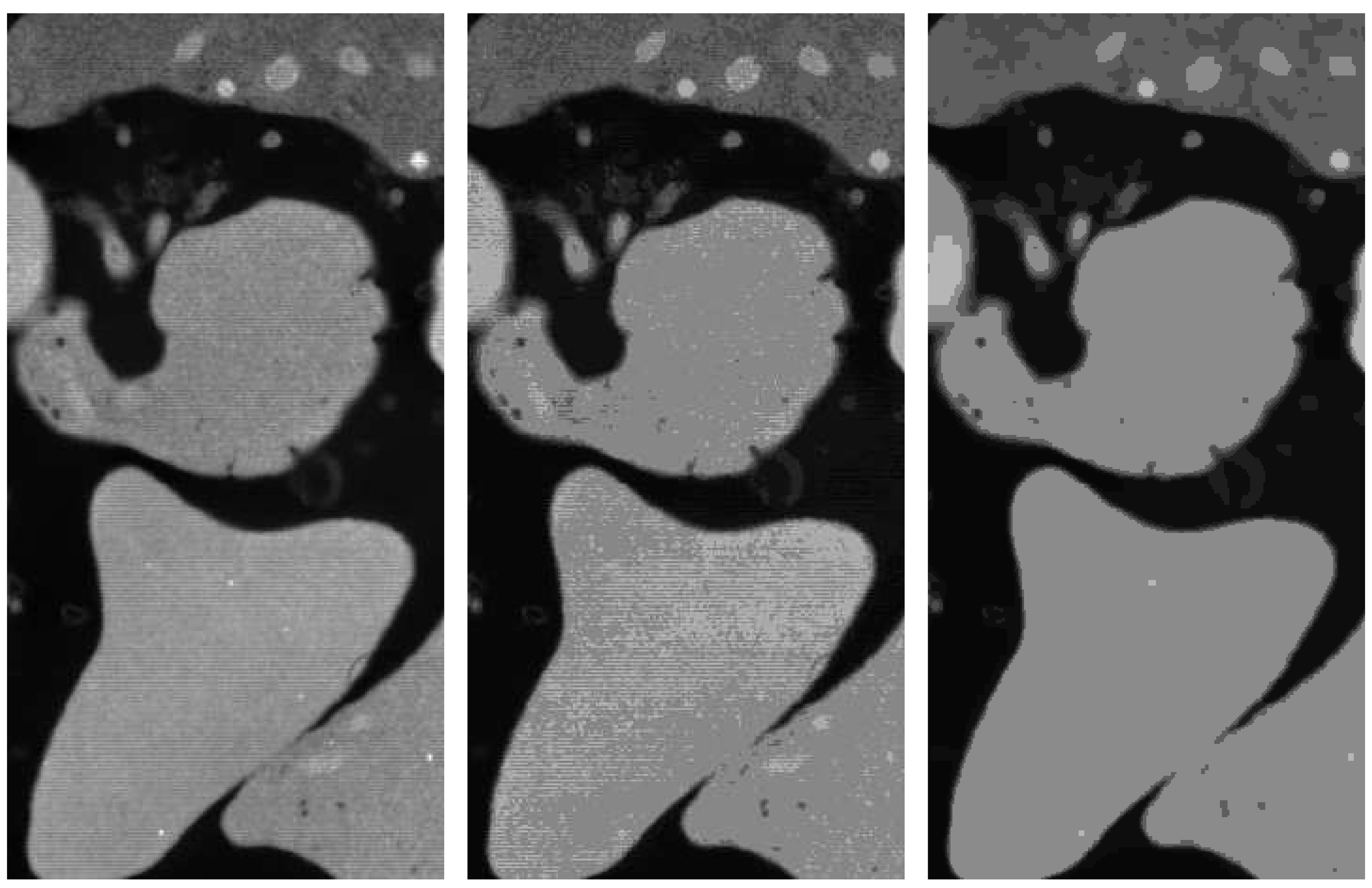
INTRODUCTION

State of the art:

✗ Lack of spatial regularity of the quantized image [Max, 60] [Lloyd, 82]

Proposed algorithm:

✓ An optimization approach involving a novel two-step, iterative, flexible, joint quantization-regularization method featuring both convex and combinatorial optimization techniques.



Original	Lloyd-Max	Proposed
		p=2, ψ=Id, μ=400
	Entropy = 0.84 bpp	Entropy = 0.56 bpp

ALGORITHM

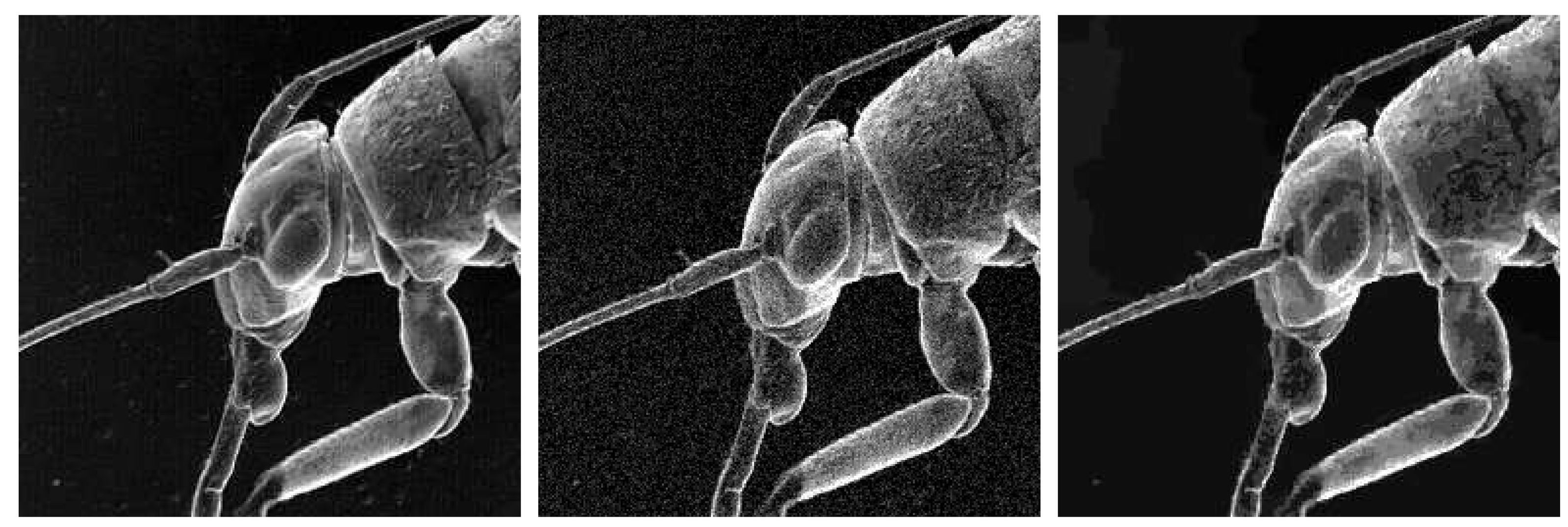
❶ Chose the number of quantization levels Q

❷ Initialize $\boldsymbol{r}^0 \in \mathbb{C}_Q$

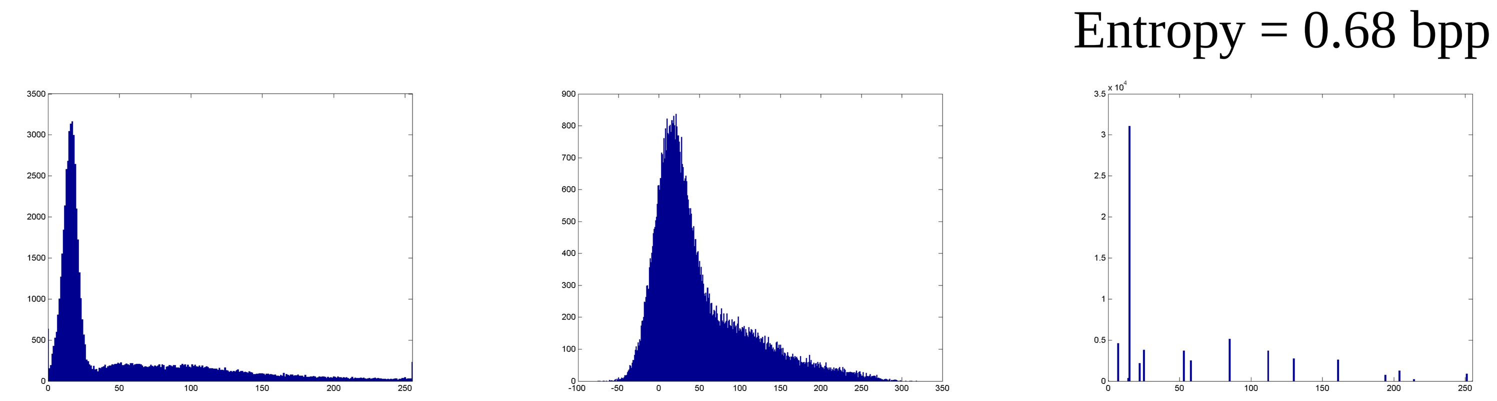
❸ **Step 1:** Find $i_{\mathcal{D}}^{\ell}$ minimizing $\varphi(q_{i_{\mathcal{D}},\boldsymbol{r}^{\ell}}, f) + \rho(i_{\mathcal{D}})$

❹ **Step 2:** Find $\boldsymbol{r}^{\ell+1}$ minimizing $\varphi(q_{i_{\mathcal{D}}^{\ell},\boldsymbol{r}}, f)$ s.t. $\boldsymbol{r} \in \mathbb{C}_Q$

Energy minimization achieved by repeating Steps 1 and 2 in the loop.



Original	Noisy Gaussian SNR = 11.4 dB	Proposed p=2, ψ=Id, μ=500 SNR = 16.3 dB Entropy = 0.68 bpp



Histograms

PROBLEM

Goal of algorithm:

- Find $\boldsymbol{r}$ - the set of Q quantization values $r_1, \dots, r_Q$.
- Assign r_i to each image position (n,m) in an *optimal* way.

Notations:

- f - an original image of size $N \times M$
- $\mathcal{D}$ - a partition of $\{1,\dots,N\} \times \{1,\dots,M\}$, $\mathcal{D} = (\mathbb{D}_k)_{1\le k\le Q}$
- $i_{\mathcal{D}}$ - a label image, $i_{\mathcal{D}}(n,m) = k \quad \Leftrightarrow \quad (n,m) \in \mathbb{D}_k$
- $q_{i_{\mathcal{D}},\boldsymbol{r}}$ - a quantized image.

Lloyd-Max quantizer:

$$\underset{i_{\mathcal{D}},\boldsymbol{r}}{\text{minimize}}\ \varphi(q_{i_{\mathcal{D}},\boldsymbol{r}}, f)$$

φ - some measure of data fidelity

Proposed quantizer:

- $\mathbb{C}_Q$ - a closed convex cone, $\mathbb{C}_Q = \big\{(s_1,\dots,s_Q) \in \mathbb{R}^Q \mid s_1 \le \dots \le s_Q\big\}$

$$\underset{i_{\mathcal{D}},\boldsymbol{r}}{\text{minimize}}\ \varphi(q_{i_{\mathcal{D}},\boldsymbol{r}}, f) + \rho(i_{\mathcal{D}}) \quad \text{s.t.} \quad \boldsymbol{r} \in \mathbb{C}_Q$$

ρ - a measure of smoothness, e.g. *anisotropic TV* defined as:

$$\rho(i_{\mathcal{D}}) = \mu\Big(\sum_{n=1}^{N-1}\sum_{m=1}^{M}\psi(|i_{\mathcal{D}}(n+1,m) - i_{\mathcal{D}}(n,m)|) + \sum_{n=1}^{N}\sum_{m=1}^{M-1}\psi(|i_{\mathcal{D}}(n,m+1) - i_{\mathcal{D}}(n,m)|)\Big), \quad \mu \ge 0$$

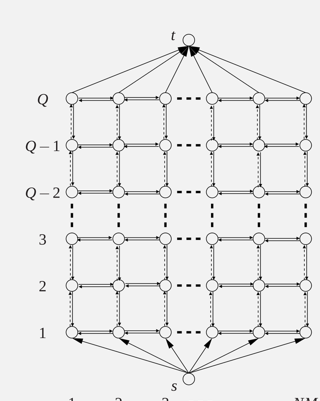
Evaluation criteria:

- SNR [dB]
- Shannon entropy of order (2,2) [bpp]

OPTIMIZATION

Step 1:

- $i_{\mathcal{D}}$ belongs to a nonconvex set of discrete values $\Rightarrow$ **nonconvex** problem $\Rightarrow$ use of **combinatorial optimization methods**
- ρ *anisotropic TV* $\Rightarrow$ **graph-cuts** : Ishikawa like framework [Ishikawa *et al.*, 99] or α-expansion algorithm [Boykov *et al.*, 01]



Ishikawa-like framework for convex functional ψ:

- labels ι take values from 1 to Q
- capacity of data edge for node $u_{\iota,j}$: $\varphi(r_\iota, f(n_j,m_j))$
- capacity of penalty edges: μ

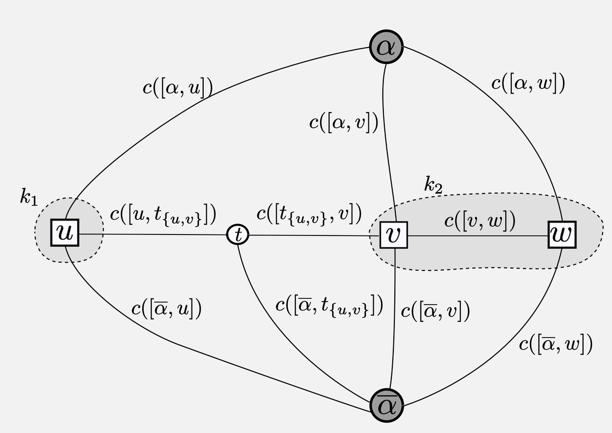


Image above:

node u belongs to label k_1
$\{v,w\}$ belong to label k_2

α**-expansion** for submodular functional ψ:

$i_{\mathcal{D}}(n_u,m_u)$ for node u is denoted by i_u

$$c([\overline{\alpha},u]) = \begin{cases} +\infty & \text{if } i_u = \alpha \\ \varphi(r_{i_u}, f(n_u,m_u)) & \text{otherwise} \end{cases}$$

$$c([\alpha,u]) = \varphi(r_\alpha, f(n_u,m_u))$$

$$\begin{cases} c([u,t_{\{u,v\}}]) = \psi(|i_u - \alpha|) \\ c([t_{\{u,v\}},v]) = \psi(|\alpha - i_v|) \\ c([\overline{\alpha},t_{\{u,v\}}]) = \psi(|i_u - i_v|) \end{cases} \quad \text{if } i_u \ne i_v$$

$$c([u,v]) = \psi(|i_u - \alpha|) \quad \text{if } i_u = i_v$$

Step 2: If $\varphi(\cdot, f)$ is convex, the determination of $\boldsymbol{r}^{(\ell+1)}$ given $i_{\mathcal{D}}^{(\ell)}$ is a conic constrained convex optimization problem.

$$\ell_p\text{-norm: } \boldsymbol{r}^{(\ell+1)} \in \underset{\boldsymbol{r}\in\mathbb{C}_Q}{\text{Argmin}} \sum_{k=1}^{Q}\sum_{(n,m)\in\mathbb{D}_k^{(\ell)}} \omega_{n,m}|r_k - f(n,m)|^p, p \ge 1$$

Solution: **proximal algorithms** [Combettes and Pesquet, 2010], **FISTA** [Beck and Teboulle, 09]