Instytut Badań Systemowych Polskiej Akademii Nauk

SEMINARIUM INSTYTUTOWE

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Shape optimization of blood flows in moving domains

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We consider the problem of shape optimization of 3D blood flows in moving domains, governed by generalized Navier-Stokes equations. To this end, we show the shape continuity of the associated solutions for a sequence of converging moving domains. After a presentation of the framework, we start showing the shape continuity of the blood velocity $\boldsymbol{u}$, for which the non-Newtonian stress is defined by:

$$\mathbf{S}(\mathrm{D}\boldsymbol{u}) := (1 + |\mathrm{D}\boldsymbol{u}|)^{q-2}\mathrm{D}\boldsymbol{u},$$

where $\mathrm{D}\boldsymbol{u} := \frac{1}{2}(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^{\mathrm{T}})$, and $q < 2$. Such fluids are called shear-thinning fluids. In [1], an existence result is provided for the case of $q > 6/5$ in moving domains, by means of the study of Generalized Bochner spaces and the Lipschitz truncation method. Thus, these techniques are used in the present case of a sequence of converging moving domains. This extends the continuity result given in [2] for $q \geq 11/5$ to the case of $q > 6/5$.

Using the classical method of calculus of variations, these results allow to show the existence of minima for a class of shape optimization problems of blood flows.

In particular, this can apply to a hemolysis minimization problem (minimization of the destruction of red blood cells).

The lack of uniqueness of shear-thinning fluids solutions prevents the study of shape sensitivity from being pursued, so that an extension of this work with computation of a shape gradient must somehow consider a regularization of the present model.

Concluding remarks will be made in this regard.

References:

[1] Philipp Nägele and Michael Ružička. Generalized Newtonian fluids in moving domains. *J. Differential Equations*, **264**(2):835–866, 2018.

[2] Jan Sokolowski and Jan Stebel. Shape optimization for non-Newtonian fluids in time-dependent domains. *Evol. Equ. Control Theory*, **3**(2):331–348, 2014