Computer Modeling

Basic notions and tools

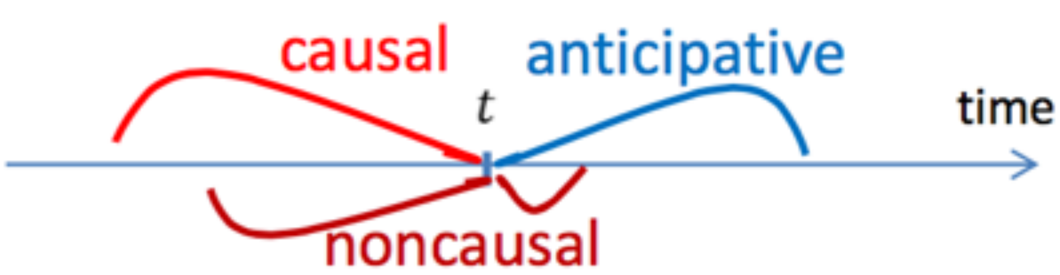
Examples

Static and dynamic systems

Suppose $\boldsymbol{G}(\boldsymbol{z}(t)) = \boldsymbol{0}$ can be solved for $\boldsymbol{y}(t)$, i.e. there exists $\boldsymbol{y}(t) = \boldsymbol{H}(\boldsymbol{u}(t), \boldsymbol{v}(t))$. Suppose also that $v(t) = 0$.

A system is static (memoryless), if the current value of the output $y(t)$ depends only on the current value of the input $u(t)$.

A system is dynamic, if the output $y(t)$ depends on the input history.



A dynamic system is causal, if the outputs depend only on the previous values of their inputs.

If the outputs depend (also) on future inputs, the system is noncausal.

If the outputs depend only on the future values of inputs, the system is anticipative.

The Laplace transform

The Laplace (one-sided, unilateral) transform is a linear integral operator

$$\mathcal{L}\{u(t)\} = \int_0^\infty e^{-st} u(t)dt \stackrel{\text{def}}{=} U(s) \stackrel{\text{def}}{=} u(s)$$

$t \geq 0$ is a real variable,
s is a complex variable

1. $u(t)$ piecewise smooth

2. $|u(t)| \leq e^{\propto t}$ for $t \to \infty$

The integral converges for $\operatorname{Re} s > \alpha$ and is an analytic function of s.

<u>Properties</u>

$a > 0, b$ – real values

1. $\mathcal{L}\{u(at)\} = \frac{1}{a} U\left(\frac{s}{a}\right)$ scaling

2. $\mathcal{L}\{u(t-a)\} = e^{-as}U(s)$ shift*

3. $\mathcal{L}\{e^{-bt}u(t)\} = U(s+b)$ modulation

4. $\mathcal{L}\{u_1(t) * u_2(t)\} = U_1(s)U_2(s)$

$$u_1(t) * u_2(t) = \int_0^t u_1(\tau)u_2(t-\tau)d\tau$$

is called the convolution $|u_1(\tau)| < M$ or $|u_2(\tau)| < M$

The Laplace transform

Przykład 1. Korzystając z definicji, wyznaczyć transformatę Laplace'a funkcji $f(t) = e^{at}$, gdzie a jest liczbą dowolną, $a \neq 0$.

The Laplace transform

Przykład 2. Korzystając z definicji, wyznaczyć transformatę Laplace'a funkcji $f(t) = \sin(at)$, gdzie a jest liczbą dowolną, $a \neq 0$.

The Laplace transform

Przykład 3. Wyznaczyć odwrotną transformatę Laplace'a

$$F(s) = \frac{s-2}{s(s-1)(s+3)}$$

The Z-transform operator: discrete-time systems

A sequence

$\{u_n\} = u_0, u_1, \dots, u_n, \dots$

The Z-transform (single-sided, unilateral) is a linear operator

$$\mathcal{Z}\{u_n\} = \sum_{n=0}^{\infty} \frac{u_n}{z^n} \overset{\text{def}}{=} U(z)$$

if the series converges.

For $u_n = 1, \quad n = 0, 1, 2, \dots$

$\mathcal{Z}\{u_n\} = \sum_{n=0}^{\infty} \frac{1}{z^n}$ geometrical series.

It converges for $\left|\frac{1}{z}\right| < 1, \ |z| > 1.$

Generally, converges outside a circle

$$|z| > R, \qquad 0 \le R \le \infty$$

Properties

1. $\mathcal{Z}\{u_{n-k}\} = z^{-k}U(z)$
2. $\mathcal{Z}\{\sum_{k=0}^{n-1} u_k\} = \frac{1}{z-1}U(z),$

   $|z| > \max(1, R)$
3. $\mathcal{Z}\{nu_n\} = -zU'(z)$
4. $\mathcal{Z}\left\{\frac{u_n}{n}\right\} = \int_0^z U(x)dx$

4. <u>Examples</u>

$$\delta_n = \begin{cases} 1 & \text{for} \quad n = 0 \\ 0 & \text{for} \quad n \neq 0 \end{cases} \qquad \mathcal{Z}\{\delta_n\} = 1$$

$$1_n = \begin{cases} 1 & \text{for} \quad n \ge 0 \\ 0 & \text{for} \quad n < 0 \end{cases} \qquad \mathcal{Z}\{1_n\} = \frac{z}{z-1}$$

The Z-transform operator: discrete-time systems

Przykład 4. Korzystając z definicji, wyznaczyć transformatę Z funkcji:

$$f(n) = \mathbb{1}(n)$$

The Z-transform operator: discrete-time systems

Przykład 4. Korzystając z definicji, wyznaczyć transformatę Z funkcji:

$$f(n) = a^n$$