Journal of ELECTRICAL ENGINEERING, VOL. 55, NO. 3-4, 2004, 109–112

COMMUNICATIONS

TIME SUB–OPTIMAL CONTROL STRUCTURES

Piotr Kowalski — Karol Krawiec *

Practical concepts for time sub-optimal feedback controllers are presented in this paper, thanks to the generalization of a classical switching curve to a switching region. Two robust structures, the so-called hard and soft ones, have been worked out and described with details. Two aspects of engineering practice are investigated: the issue of velocity limitation and the generalization of a target set to any point in the state space, especially including cases where a target position must be reached with a given velocity.

K e y w o r d s: time-optimal control, suboptimal feedback controler

1 INTRODUCTION

In this paper practical concepts for time sub-optimal feedback controllers are presented by generalization of classical switching curves to the switching region. The time to reach the target set is the shorter the more precise is the identification of the probabilistic data on the object. In presented considerations two problems - aspects of engineering practice - are investigated: the issue of velocity limitation and generalization of the target set to any point in state space, especially including cases where the target must be reached with a given velocity.

2 CLASSICAL TIME OPTIMAL CONTROL STRUCTURES

Let's define:
Time optimal control as a control which brings the object from starting point x_0 to finishing point x_k with minimal finite time t_{opt}.
Time sub-optimal control as a control which brings the object from starting point x_0 to finishing point x_k with finite time $t : t > t_{opt}$ but $t \simeq t_{opt}$.
In the modern control engineering a model can be described in state space:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{1}$$

where: u is a control vector, x is a state vector and A, B, C, D are matrices describing model.

Let's define model:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u(t) \end{cases} \quad \text{and} \quad |u(t)| < 1 \tag{2}$$

Optimal solutions of (2):
$u(t) \in \{+1\}, \{-1\}, \{+1 -1\}, \{-1 +1\}$ with initial conditions:

$$\begin{cases} x_1(0) = \xi_1 \\ x_2(0) = \xi_2 \end{cases} \tag{3}$$

Solution of presented state space model:

$$\begin{cases} x_1 = \frac{\Delta t^2}{2} + \xi_2 t + \xi_1 \\ x_2 = \xi_2 + \Delta t \end{cases} \tag{4}$$

where: $\Delta = u_{opt}(t) = \pm 1$

$$t = (x_2 - \xi_2) \tag{5}$$

$$x_1 = \frac{\Delta x_2^2}{2} - \frac{\Delta \xi_2^2}{2} + \xi_1 \tag{6}$$

Solution of trajectories equation (6) is presented in Fig. 1.

γ curves

A γ_+ curve is the locus of (x_1, x_2) points (states) which can be brought to $(0,0)$ point with the control $u = 1$, $\gamma_+ = \{(x_1, x_2) : x_1 = \frac{1}{2}x_2^2, x_2 \le 0\}$.

A γ_- curve is the locus of (x_1, x_2) points (states) which can be brought to $(0,0)$ point with the control $u = -1$, $\gamma_- = \{(x_1, x_2) : x_1 = -\frac{1}{2}x_2^2, x_2 \ge 0\}$.

A γ curve (Fig. 2), is defined as the switching curve, $\gamma = \{(x_1, x_2) : x_1 = \frac{1}{2}x_2|x_2|\} = \gamma_+ \cup \gamma_-$

R sets

A R_+ is set of points (Fig. 2) in states space (x_1, x_2) where: $R_+ = \{(x_1, x_2) : x_1 < \frac{1}{2}x_2|x_2|\}$.

A R_- is set of points (Fig. 2) in states space (x_1, x_2) where: $R_- = \{(x_1, x_2) : x_1 > \frac{1}{2}x_2|x_2|\}$.

* Faculty of Electrical and Computer Engineering, Cracow University of Technology ul.Warszawska 24, 31-155 Cracow, Poland E-mails: pkowal@twins.pk.edu.pl, karol.krawiec@wp.pl

ISSN 1335-3632 © 2004 FEI STU

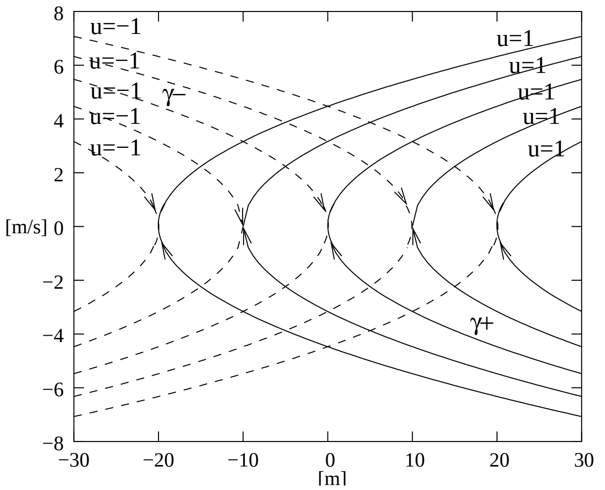

Fig. 1. Trajectories in equation (6)

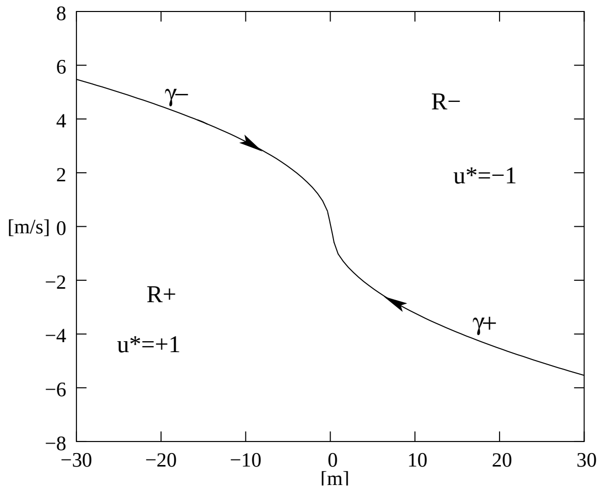

Fig. 2. Illustration of γ_+ and γ_- curves

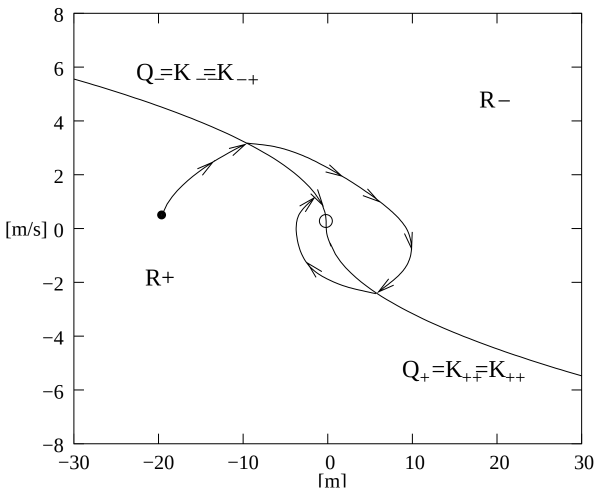

Fig. 3. Oscillations around the target set

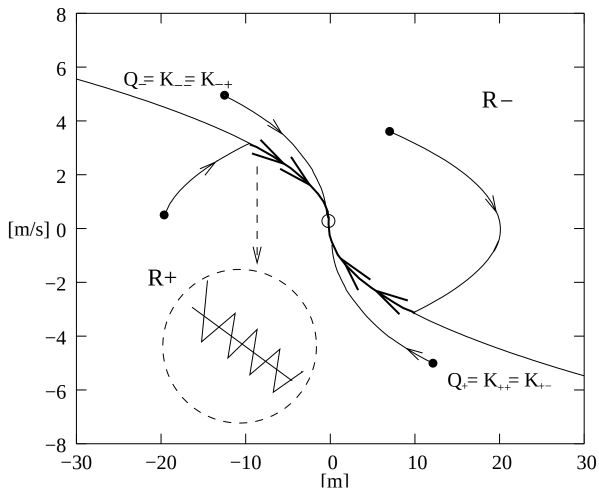

Fig. 4. Sliding trajectories

3 PRACTICAL MODEL OF TIME OPTIMAL CONTROL

In the engineering practice while considering the issue of the resistance to motion in the object, three different cases may occur in the model of control. The first case is when the parameter of motion resistance v is equal to the assumed value v' used in the feedback controller equations. Parameter v' is an estimate of real value of motion resistance and it can be interpreted as an indefinite knowledge about the parameter v needed for the purpose of designing feedback controller. When v' is equal to v, the control is time optimal. The state of the system is brought to the switching curve being permanently included in this curve. It reaches the target set in a minimal and finite time. The second case - when the value of estimate assumed in the controller is greater than value of motion resistance is presented in Fig. 3. As a result of the oscillations around the target, overshoot occur in the system. Target is reached in a finite time. The switching region Q is confined to the curve whose shape is dependent on the parameter v, describing the resistance to motion. Sets K_{+-} and K_{++} represent all those states which can be brought to the origin of coordinates by the control +1, at the minimum and maximum values of motion resistance respectively. The set Q_+ contains intermediate points. The sets K_{--}, K_{-+} and Q_- for the control -1 may be interpreted in the same way. The third case when value of parameter v' is lower than the real value v, respective trajectories are presented in Fig. 4. After the switching curve is crossed sliding trajectories appear in the system. The target set is reached in a finite time. In each case the time to reach the target set increases with respect to the optimal more or less proportionally to the difference between the values v' and v.

4 APPLICATION CONCLUSIONS – SUBOPTIMAL CONTROL STRUCTURE I

In practice the value of parameter v is not know a priori. In our considerations it will be treated as a random

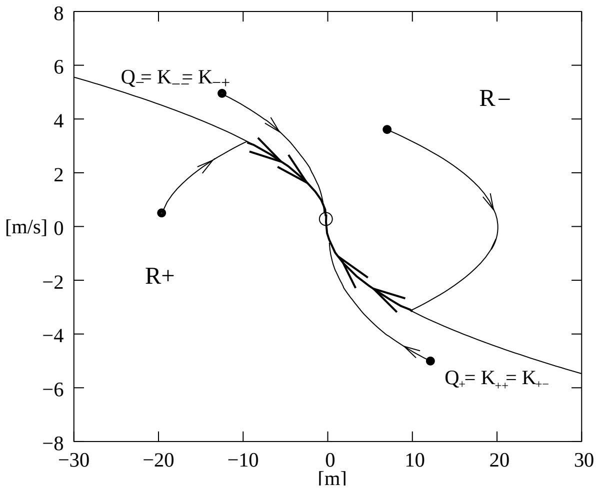

Fig. 5. Trajectories generated by structure I

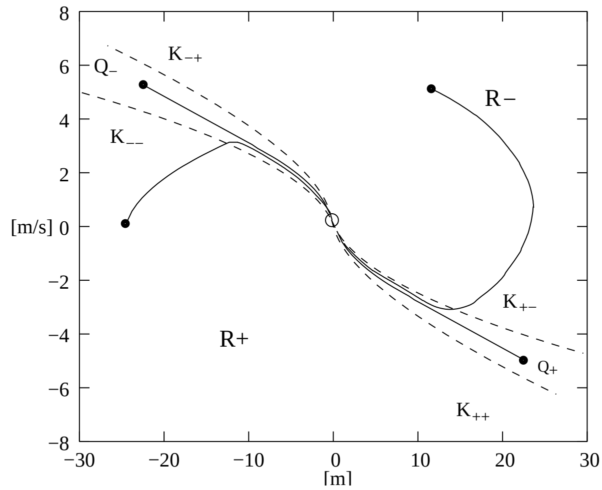

Fig. 6. Trajectories generated by structure II

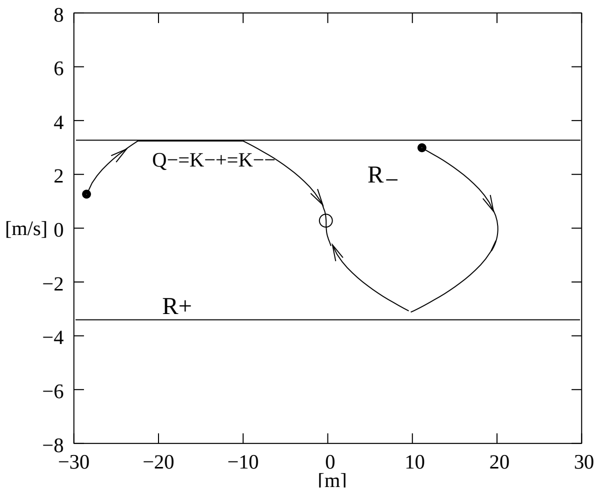

Fig. 7. Trajectories with limited velocity

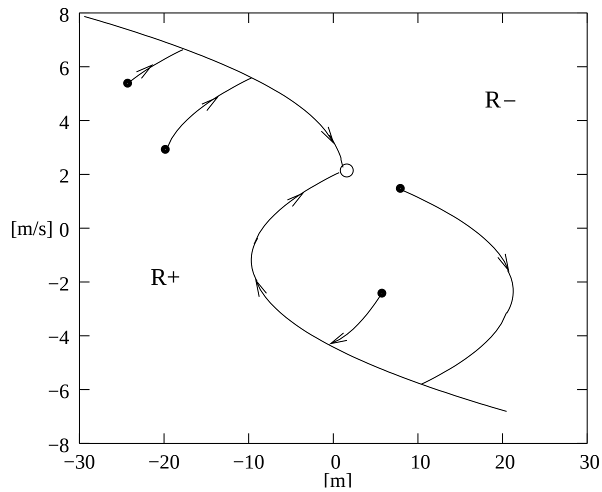

Fig. 8. Trajectories with generalization of target set

variable. The value of its estimate v' will be treated as a decision whereas the parameter v will be considered as a state of nature. The first example is based on physical properties of friction phenomena and the fact that the influence of resistance to motion on dynamical processes can be averaged. Having the value of estimate v' the feedback controller equations can be calculated. Equations of the switching curve are defined on the $x_1 - x_2$ plane according to (7):

$$\begin{aligned} x_1 &= \tfrac{x_2^2}{2(1+\nu')} \text{ for } x_2 \in (-\infty, 0) \\ x_1 &= -\tfrac{x_2^2}{2(1+\nu')} \text{ for } x_2 \in (0, \infty) \end{aligned} \tag{7}$$

Fig. 5 is an illustration of the control structure and the trajectories it generates. The suboptimal control can be defined in a closed-loop manner by equality:

$$U_s = \begin{cases} -1 & if \quad (x_1, x_2) \in (R_- \cup Q_-) \\ 0 & if \quad (x_1, x_2) \in [0, 0] \\ +1 & if \quad (x_1, x_2) \in (R_+ \cup Q_+) \end{cases} \tag{8}$$

The control designed above may lead to chattering - frequent switchings between the two values +1 and -1 along sliding trajectories should be avoided. This feature can have the negative impact on the endurance of a device and a user comfort. Under the condition that the control may take any value in the interval [-1,+1] the second control structure can be considered.

5 APPLICATION CONCLUSIONS – SUBOPTIMAL CONTROL STRUCTURE II

Previous structure can be modified by introducing continuous instead of discontinuous control law. In addition to the constant v' introduced in the structure I, let the parameter v'' also be given with the condition $-1 < v'' < v'$. Let K_{+-} and K_{++} denote sets of all states which can be brought to the origin by the control +1 if $v = v''$ or $v = v'$ respectively. In the same way sets K_{-+} and K_{--} can be defined. The suboptimal control can be defined by the formula (9) where the function

$z(x_1, x_2)$ takes on the value $1 - v_{++}v_{-}$ on the sets K_{+-} and K_{--}, after that it increases linearly to the value 1 on sets K_{++} and K_{-+}, according to the formula (10). A suitable value for the parameter v'' can be determined heuristically - in general the difference $v'' - v'$ should be proportional to the delay in the system.

$$U_s = \begin{cases} -1 & \text{if} \quad (x_1, x_2) \in R_- \\ -z(-x_1, -x_2) & \text{if} \quad (x_1, x_2) \in Q_- \\ 0 & \text{if} \quad (x_1, x_2) \in [0, 0] \\ z(x_1, x_2) & \text{if} \quad (x_1, x_2) \in Q_+ \\ 1 & \text{if} \quad (x_1, x_2) \in R_+ \end{cases} \tag{9}$$

$$z(x_1, x_2) = \frac{v_+ - v_-}{-k_{++}(x_2) - -k_{+-}(x_2)}(x_1 - k_{++}(x_2)) + 1 \tag{10}$$

Trajectories generated by this control formula are shown on Fig. 6. They resemble the results achieved on a bobsled track thanks to the appropriate modeling of its shape.

6 GENERALIZATIONS OF SUBOPTIMAL CONTROL STRUCTURES

This section presents some generalizations of suboptimal control structures. The first example to be considered will be the issue of velocity limitation, often essential in many engineering applications. Without this aspects the basic time optimal control law for a mechanical system - for example a robot manipulator - may lead to unacceptably high velocities if the distance between the initial and target point is too long. Let's take into account the condition of limiting the velocity to the value b by introducing the assumption $|x_2| < b$ (Fig. 7).

The second example to be considered is the generalization of the target set to any point in the state space especially including cases where the target position must be reached with a given velocity. The Fig. 8 illustrates the trajectories generated by control from structure I with its modification. If second coefficient in a state space vector is equal to zero then the presented closed-loop structures remain the same - due to possibility of performing the simple transformation. Only in the case where the target position must be reached with a given velocity it becomes necessary to introduce a modification which is the one of our future researches.

7 SUMMARY

In this paper several practical concepts for time suboptimal control structures were introduced. It should be strongly emphasized that the presented control structures turn out to be slightly sensitive to the inaccuracy resulting from identification. Such kind of robustness should be considered as a very valuable property of this control systems.

Acknowledgement

The authors acknowledge for helpful suggestions of Piotr Kulczycki DSc.

References

[1] P. Kulczycki.: Fuzzy Controller for Mechanical Systems, IEEE Transactions on Fuzzy Systems, vol. 8, 2000, pp. 645–652.

Received 1 June 2003

Piotr Kowalski, born in Cracow, an undergraduate student of Teleinformatic at the Faculty of Electrical and Computer Engineering at the Cracow University of Technology. His supervisor is DSc Piotr Kulczycki. He is interested in microprocesor technology, control systems and neural networks.

Karol Krawiec, born in Cracow, an undergraduate student of Teleinformatic at the Faculty of Electrical and Computer Engineering at the Cracow University of Technology. His supervisor is DSc Piotr Kulczycki. He is interested in control systems and software engineering.