

Optimal control and Hamiltonian dynamics

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Abstract. Extremals of optimal control problems are solutions to Hamiltonian systems. In my talk I am going to show how the intuition and techniques of Optimal Control Theory help to study Hamiltonian Dynamics itself; in particular, to obtain an effective test for the hyperbolicity of invariant sets and to find new systems with the hyperbolic behavior.

Bang-bang control in the singular perturbations limit

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Abstract. Singularly perturbed control systems for which the bang-bang principle holds are considered. The limit behavior of bang-bang controls is examined and compared with the corresponding controls of the variational limit.

Variational theory of optimal taxation

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Abstract. The theory of income taxation leading to problems of calculus of variations and optimal control will be reviewed. In particular, we will discuss the 1897 Edgeworth concept of *sum total happiness* and *equal sacrifice* tax, inverse theorems of the latter, as well as the "Nobel" prized works of Vickerey and Mirrlees focusing on incentives and assymmetric information.

New necessary conditions in optimal control

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Abstract. We present new necessary conditions of optimality for a general control problem formulated in terms of a differential inclusion. These conditions constitute a new state of the art, subsuming, unifying, and substantially extending the results in the literature. The Euler, Weierstrass and transversality conditions are expressed in their sharpest known forms. No assumptions of boundedness or convexity are made, no constraint qualifications imposed, and only weak pseudo-Lipschitz behavior is postulated on the underlying multifunction. The conditions also incorporate a '*stratified*' feature of a novel nature, in which both the hypotheses and the conclusion are formulated relative to a given radius function. When specialized to the calculus of variations, the results yield necessary conditions and regularity theorems that go significantly beyond the previous standard. They also apply to parametrized control systems, giving rise to new and stronger maximum principles of Pontryagin type.

Nonlocal metric regularity of nonlinear operators

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Abstract. Let $F : X \rightarrow Y$ be a mapping between Banach spaces. It is known that if $F'(x_0)$ is onto, then F is metric regular (covers with a linear rate) in a neighborhood of x_0 . We are interested to find conditions, under which this property holds on a broader set, not just locally. Such conditions are based on the uniform covering of the derivative $F'(x)$. In the talk we will give some general facts about this, and point out a special class of operators in functional spaces, for which the uniform covering, though not following from the general theorems, can be however obtained from its specificity. This class appears in the study of convexification of a general nonlinear control system (i.e., a system with sliding modes), and the obtained nonlocal metric regularity is used as a key tool in the proof of the corresponding approximation theorem that connects solutions of the relaxed (convexified) and initial systems.

On the global asymptotic stability problem and the Jacobian conjecture

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Abstract. In this survey, we recall the formulation of the problems and give a review of some nontrivial results in the area. Let $F = (F_1, \dots, F_n) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be a C^1 map and let $F'(x)$ and $\text{Jac } F(x) = \det F'(x)$ denote the Jacobian matrix and the jacobian of F at a point $x \in \mathbb{R}^n$, respectively. The Global Asymptotic Stability Problem (GASP) reads as follows: Assume that $F(0) = 0$ and at any point $x \in \mathbb{R}^n$ all eigenvalues of $F'(x)$ have negative real parts. Then consider the associated system of differential equations $x'_j(t) = F_j(x_1(t), \dots, x_n(t))$, $j = 1, \dots, n$. The question is whether the solution $x(t) = 0$ is globally asymptotically stable. If $n > 2$, then the answer is negative (even if F is a polynomial automorphism), so from now on (GASP) denotes (GASP) restricted to \mathbb{R}^2 . In 1963, Olech showed that under the (GASP) assumption (i. e., $\text{Jac } F(x) > 0$ and $\text{Trace } F'(x) = \frac{\partial F_1}{\partial x_1}(x) + \frac{\partial F_2}{\partial x_2}(x) < 0$ for any $x \in \mathbb{R}^2$) the conclusion of (GASP) is equivalent to the injectivity of F . In 1994, Fessler, and independently Gutierrez, proved the injectivity of F and, due to the above mentioned Olech's equivalence, gave the affirmative answer to the two-dimensional (GASP).

Let \mathbb{K} denote \mathbb{R} or \mathbb{C} , $n > 1$. The Jacobian Conjecture can be formulated as follows: If $F = (F_1, \dots, F_n) : \mathbb{K}^n \rightarrow \mathbb{K}^n$ is a polynomial map with a constant nonzero jacobian, then F is a polynomial automorphism (i. e., there exists F^{-1} and F^{-1} is also a polynomial map). Although the Jacobian Conjecture is still unsolved even in the case $n = 2$, it is convenient to consider the so called Generalized Jacobian Conjecture (for short (GJC)): *the Jacobian Conjecture holds for every $n > 1$* . We give a review of some interesting conditions equivalent to the Jacobian Conjecture, including Meisters and Olech's result on the existence of a poly-flow solution of the associated Ważewski equation $x'(t) = [F'(x(t))]^{-1}(a)$. We also present a reduction of (GJC) to the case of F of degree 3 and of special forms, then some partial results, and (JC)'s relations with other problems.

On stability properties of optimal bang-bang controls for linear and semilinear systems

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Keywords: optimality conditions, stability in optimal control, optimality conditions, bang-bang structure stability.

Abstract. Optimal control problems for systems linearly depending on the control input often admit bang-bang type solutions where control takes exclusively extremal values. Over the last years substantial results on optimality conditions and the solution structure stability have been obtained.

In the paper, we analyze once more the related finite-dimensional program with switching points localizations taken as decision variables. For linear as well as semilinear systems, we find an explicit representation for the Hessian w.r.t. switching times and deduce sufficient optimality conditions. The criteria are compared to duality based optimality results which are particularly simple for the linear system case.

Further, certain multi-point boundary value problems are derived for the calculation of sensitivity differentials, and appropriate bang-bang controllability assumptions are discussed.

Regularity of the boundary of reachable sets and semiconcavity of the value function

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Abstract. In this paper, first we investigate the attainable set in time T for the control system $\dot{y}(t) = f(y(t), u(t))$, obtaining sufficient conditions for such a set, to satisfy a uniform interior sphere condition for $T > 0$. Then, we apply our analysis to obtain a semiconcavity property for the value function of time optimal control problems with a general target, and to deduce a $C^{1,1}$ -regularity result for boundaries of attainable sets.

Decomposable mappings and integral functionals

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Abstract. A mapping $k : L^1(T, X) \rightarrow L^1(T, R)$ will be called decomposable iff for any $u, v \in K$ one has

$$k(\chi_A u + (1 - \chi_A)v) \leq \chi_A k(u) + (1 - \chi_A)k(v).$$

Equivalently, the L -epigraph

$$K = \{(u, v) \in L^1(T, X \times R) : v(t) \geq k(u)(t) \text{ a.e. in } T\}$$

is decomposable.

To each decomposable mapping $k : L^1(T, X) \rightarrow L^1(T, R)$ corresponds an integral functional given by

$$\mathbf{I}(u) = \int_T k(u)\mu(dt).$$

There are given necessary and sufficient conditions for lower semicontinuity of the functional $\mathbf{I}(u)$ in a certain weak topology in $L^1(T, X)$. The result is based on the analysis of the weak closure of the L -epigraph K .

Discovery of the maximum principle

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Abstract. A short history of the discovery of the maximum principle in optimal control by L.S. Pontryagin and his associates is presented.

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Differential inclusions with unbounded values: regularity, existence and relaxation

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Abstract. The main questions to be discussed concerns regularity properties of certain mappings associated with differential inclusions. As a consequence we obtain extensions of Filippov's classical existence and relaxation theorems to unbounded differential inclusions satisfying weaker Lipschitz-type requirements.

On the role of singular extremals

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Abstract. Singular extremals are curves which satisfy the Pontryagin Maximum Principle with the Lagrange coefficient at the cost function equal to zero. Thus, they depend on the control system, only. We will discuss their role in the geometry of the system and, in particular, possibility of constructing feedback invariants based on them.

Nonlinear wave equations with nonmonotone, degenerate damping and source terms

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Abstract. We consider the following model of a nonlinear wave equation described by differential inclusion:

$$u_{tt} - \Delta u + |u|^k \partial j(u_t) \subset |u|^{p-1} u; \quad (1)$$

and defined on a bounded domain $\Omega \times [0, T]$, where $\Omega \subset R^n, n \geq 2$.

∂j denotes a sub-differential of a continuous, convex function j and the parameters k, p are positive. Solutions u are considered subject to either Dirichlet or Neumann boundary conditions and finite energy $H^1 \times L_2$ initial conditions.

Models with $k = 0$ and $\partial j(s) = |s|^{m+1}, m > 0$ have recently attracted considerable attention in the literature. In fact, for these special cases an almost complete theory of existence and nonexistence of solutions is already in place. Depending on the relation between parameters m and p , which represent relation of the damping to the source, results pertaining to global solvability and finite time blow-up of solutions became available in the literature.

The main distinct feature of the model under consideration, which is motivated by several applications in quantum field theory and solid mechanics, is non-monotone character of the damping. Indeed, when $k > 0$, the damping term which can be degenerate is no longer represented by a monotone operator. Thus, the usual tools of monotone operator theory are no longer applicable to the study of this class of non-monotone problems.

The main aim of this talk is to discuss recent results displaying an interplay between the degenerate damping and a source for the model in (1). In particular, we shall present results on (i) global existence of finite energy generalized solutions, (ii) formation of singularities in a finite-time, and (iii) propagation of regularity.

This is joint work with Viorel Barbu.

Sensitivity analysis and real-time control for optimal bang-bang control problems

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Abstract. We study bang-bang control problems that depend on a parameter p . For a fixed nominal parameter p_0 , it is assumed that the bang-bang control has finitely many switching points and satisfies second order sufficient conditions (SSC). SSC can be expressed and checked in terms of an associated finite-dimensional optimization problem where the switching points, the free final time and the unknown components of the initial state are considered as optimization variables. The theory of SSC will be discussed in greater detail in the talk of Nikolai Osmolovskii.

We show that the nominal optimal bang-bang control can be locally embedded into a parametric family of optimal bang-bang controls where the switching points are differentiable function of the parameter. A well known sensitivity formula from optimization is used to compute the parametric sensitivity derivatives of the switching points and free final time. This approach also allows to determine the sensitivity derivatives of the optimal state trajectories. Real-time control strategies for perturbed bang-bang controls then are easily implementable on the basis of sensitivity derivatives of switching points. These techniques are illustrated by several numerical examples from robot control, nonlinear optics, chemical engineering and economics. The talk is based on a joint work with Christoph Büskens, Ralf Hannemann, Jang-Ho Robert Kim and Yalcin Kaya.

Optimal control of evolution and partial differential inclusions

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Abstract. This talk is devoted to optimal control problems governed by evolution/differential inclusions in infinite-dimensional spaces and also by semilinear partial differential inclusions. We pursue a twofold goal: to develop the method of discrete approximations for such problems and to derive necessary optimality conditions of the Euler-Lagrange type under natural assumptions. First we consider a generalized Bolza problems for infinite-dimensional differential inclusions with endpoint constraints. One of the principal differences between finite-dimensional and infinite-dimensional dynamic systems is the lack of compactness in infinite dimensions. Constructing well-posed discrete approximations and using advanced tools of variational analysis and generalized differentiation, we derive necessary conditions for discrete-time problems and then, by passing to the limit, for continuous-time evolution inclusions. A similar procedure is developed for optimal control problems of the Mayer type governed by constrained semilinear inclusions with unbounded operators generating compact semigroups. This particularly covers parabolic partial differential inclusions whose solutions are understood in the conventional mild sense.

On equivalence of second order optimality conditions for bang–bang control problems

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Abstract. Second order necessary and sufficient optimality conditions for bang-bang control problems have been originally studied in Milyutin, Osmolovskii (1998). These conditions amount to testing the positive (semi-)definiteness of a quadratic form (associated with certain Lagrange multipliers) on a critical cone. A direct numerical verification of this test can be carried out only in some special cases. Therefore, in Maurer, Osmolovskii (2001,2003) we studied several representations of the critical cone and transformations of the quadratic form such as to obtain more practical second order conditions. The second order test has been successfully applied to several numerical examples, representing different types of bang-bang control problems.

In the present talk we are interested in exploring the relations between the second order sufficient conditions in Milyutin, Osmolovskii (1998) and in Agrachev, Stefani, Zezza (2002). A careful study of the second order variations of state trajectories w.r.t. switching points of control and initial value of state reveals that the conditions in Milyutin, Osmolovskii (1998) and in Agrachev et al. (2002) are indeed equivalent under the assumption of normality. Our study leads to some generalizations of the results in Agrachev et al. (2002). Moreover, we obtain a more explicit representation of the quadratic form used for checking second order conditions. The talk is based on a joint work with Helmut Maurer.

Differentiability and inversion of multimappings and differential inclusions

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Abstract. The extension of the classical inverse mapping theorem to correspondences (or relations, or multifunctions or multimappings) has been the object of numerous works. The variety of concepts and methods involved corresponds to the multiplicity of purposes (mathematical programming, sensitivity analysis, variational inequalities...). Here our method relies on a fixed point theorem, as in the classical case of mappings. Thus, it is rather simple; it is not as constructive as the classical Newton scheme, but the Nadler fixed point theorem it uses also involves an iteration process. As an application, we extend the approach of Robbin [10] to existence results for ordinary differential equations to existence results for differential inclusions.

This is joint work with Khadra Nachi.

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Approximation of the value functions of control systems with state discontinuous dynamics

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Abstract. The value function in the Mayer problem with dynamics given by an upper semicontinuous set-valued map satisfies the dynamic programming principle. But it is not a lower semicontinuous solution of the corresponding Hamilton-Jacobi-Bellman equation (solution in the meaning of Frankowska (SIAM J. Control Opt 1993)). However, an upper semicontinuous map can be approximated by Lipschitz continuous maps. The approximation scheme can be transferred by the Fenchel transform onto Hamiltonians. This procedure suggests how to define a solution to the Hamilton-Jacobi equation with a discontinuous convex Hamiltonian. The approximation scheme will be checked in several optimal control problems.

On extensions of Ważewski topological principle to differential inclusions

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Keywords: Viability theory, differential inclusions, topological methods.

Abstract. We study the existence of trajectories to differential equations or inclusions remaining in a given compact subset of R^n . Topological properties of the boundary of this set enables us to conclude the existence of such trajectories in several cases.

Bifurcations of control-affine systems in the plane

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Abstract. We define bifurcations of control-affine systems in the plane and classify all generic 1-parameter bifurcations with nonvanishing control vector field. More precisely, we attach to a planar system three invariants of usual feedback equivalence, which are the equilibrium set, the set of time-critical trajectories and the foliation of the trajectories of the control vector field. We classify topological bifurcations of those three invariants. There are six such bifurcations: two bifurcations of equilibrium sets, two bifurcations of critical sets and two bifurcations of pairs of invariants. We also analyze how time optimality properties and stabilizability of generic planar families change when the parameter varies. Finally, we analyze all bifurcations of planar systems with quadratic nonlinearities, in particular, those not appearing among generic ones.

This is joint work with B. Jakubczyk.

Paraconvex analysis

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Abstract. Let $\alpha(t)$ be a nondecreasing function mapping the interval $[0, +\infty)$ into the interval $[0, +\infty]$ such that

$$\lim_{t \downarrow 0} \frac{\alpha(t)}{t} = 0.$$

We say that the function $f(\cdot)$ is *strongly $\alpha(\cdot)$ -paraconvex* if there is a constant $C > 0$ such that for all $x, y \in \Omega$ and $0 \leq t \leq 1$

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) + C \min[t, 1-t] \alpha(\|x-y\|).$$

The aim of this talk is to show

Theorem . *Let Ω be an open convex set of a real Banach space X . Suppose that X is an Asplund space (in particular it has separable dual X^*). Let $f(\cdot)$ be a strongly $\alpha(\cdot)$ -paraconvex function. Then there is a set of the first category $A_F \subset \Omega$ such that the function $f(\cdot)$ is Fréchet differentiable at every point $x_0 \in \Omega \setminus A_F$.*

Controlled Navier-Stokes and Euler equation: controllability and accessibility by geometric control methods

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Abstract. We present results of joint work with A.A.Agrachev (SISSA, Trieste, Italy) which regards controllability issues for 2D Navier-Stokes (NS) and Euler systems, describing the movement of respectively incompressible viscous and ideal fluid. Admissible controls are taken from small-dimensional functional space. In our previous work ([1, 2, 3] we studied: conditions for global controllability of finite-dimensional Galerkin approximations of 2D and 3D NS and Euler systems, conditions for global controllability in finite-dimensional projection of 2D NS system and conditions for L_2 -approximate controllability for 2D NS system. Our study of controllability exploits exclusively nonlinear terms; differential geometric/Lie algebraic methods and relaxation techniques are intensively used.

In our current work we study property of accessibility in observed projection for 2D NS system and provide Lie algebraic approach to establishing accessibility criteria. We also provide an alternative approach for establishing controllability in observed projection.

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Singular perturbations and optimality conditions in shape optimization

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Abstract. Selfadjoint extensions of elliptic operators are used in modelling of singular perturbations of geometrical domains. The first order optimality conditions are derived for a class of shape optimization problems.

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On some approximation problems related to time-discretizations of control systems

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Abstract. A number of approximation problems that arise in the time-discretization of control systems or differential inclusions will be considered. Some of them are:

- relaxation of discrete-time inclusions;
- approximation of integrals of set-valued mappings;
- approximation of the set of trajectories of a differential inclusion;
- approximation of the reachable set of a control systems;
- high order approximations to optimal control problems.

Also some open problems will be discussed.

Second order conditions in optimal periodic control: an application to the sailboat control problem

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Abstract. Optimal periodic control problems are special kinds of optimal control problems for which the initial and terminal values of the state are constrained to coincide. Such problems arise in sustainable resource economics, where the boundary conditions reflect the fact that the initial population must be the same as the terminal population to ensure sustainable harvesting. The theory of optimal periodic control also has a role to play in both mechanical and process systems control, when we seek to determine whether a cost can be reduced by periodic operation in place of steady state operation. Speyers earlier work on reducing the midcourse fuel consumption of an aircraft by periodic motion has received particular attention. We focus on the application of second order sufficient conditions of local optimality to another problem studied by Speyer, concerning optimal control strategies for a sailboat travelling into the wind. We compare the effectiveness of different second order techniques in identifying the region in parameter space for which the optimal strategy is periodic, and suggest some improvements.

Controllability and Liouville problem

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Abstract. Necessary and sufficient conditions under which a linear, infinite dimensional control system can be steered to the origin with arbitrary small energy will be given. An application of the result to a generalisation of the classical Liouville problem on harmonic functions will be presented. Its stochastic implications will be discussed as well. The talk will be based on a joint work with E. Priola.

A Hamiltonian approach to state local optimality for a bang-bang trajectory

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Abstract. We will describe sufficient conditions for a bang-bang extremal $\hat{\xi}$ to be a strong local optimum, here strong means that we consider the C^0 topology in the state space, for a control problem in the following form

$$\text{Minimize } c_0(\xi(0)) + c_T(\xi(T)) \quad (2)$$

subject to

$$\dot{\xi}(t) = f_0(\xi(t)) + u(t) f_1(\xi(t)), \quad u(t) \in [-1, 1] \quad (3)$$

$$\xi(0) \in N_0, \quad \xi(T) \in N_T, \quad (4)$$

N_0, N_T are given smooth submanifolds of M possibly reduced to a single point, and c_0, c_T are smooth real-valued functions and $f_0, f_1 : M \rightarrow TM$.

We will analyze both the cases of singular and regular extremals. In the case of bang-bang extremals, the kernel of the first variation of the problem is trivial, and hence the usual second variation, which is defined on the kernel of the first one, does not give any information. We solve this problem by considering the finite-dimensional subproblem generated by perturbing the switching times. The usual (finite-dimensional) second order optimality conditions for this problem give an appropriate second variation. Moreover, since the maximized Hamiltonian is not smooth at the switching points we need to give conditions which assure us that its flow is defined and piecewise smooth around the reference adjoint covector. Indeed we prove that the positivity of this second variation yields that the Hamiltonian flow has the properties we have described so that we can prove strong local optimality for the reference trajectory. The set of admissible variations on which we test the second variation can be very small, its dimension can be less than the state space dimension, and when it is zero, we directly have optimality.