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1 Biobjective multidimensional knapsack problems

Beasley OR library contains a set multidimensional knapsack test problems (http://people.brunel.ac.uk/∼ mastjjb/jeb/info.html, for description of the problems see Chu and Beasley 1998, or *MK_test_problems* pdf document at http://www.ibspan.waw.pl/∼kaliszew/MK_problems/).

There are 9 groups of problems with numbers of constraints m set to $5, 10, 30$, and the number of variables n set to $100, 250, 500$ and each group contains 30 problems, giving a total of 270 problems. Data are as described in Chu and Beasley (1998).

From singleobjective problems of Chu and Beasley. For each original Chu and Beasley problem we added the second objective function, coefficients of which were a random permutation of the coefficients of the first objective function. Data for the problems are organized according to the LP format.

2 A transformation of multiobjective problems to singleobjective ones

The multiobjective optimization problem is defined as:

$$\begin{gathered} vmax f(x) \\ x \in X_0 \end{gathered} \tag{1}$$

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where $f : X \to R^k,\ f = (f_1, ..., f_k),\ f_l :\to R,\ l = 1, ..., k,\ k \geq 2$, are objective functions, and $vmax$ denotes the operator of deriving all Pareto optimal solutions in X_0.

Solution $\bar{x}$ is Pareto optimal if $f_l(x) \geq f_l(\bar{x}),\ l = 1, ..., k$, implies $f(x) = f(\bar{x})$.

It is a well-established result (cf. Kaliszewski 2006, Kaliszewski et al. 2016, Ehrgott 2005, Miettinen 1999) that solution x is Pareto optimal [1] if and only if it solves the singleobjective optimization problem

$$\min_{x \in X_0} \max_l [\lambda_l (y^*_l - f_l(x)) + \rho e^k (y^* - f(x))] \tag{2}$$

where $\lambda_l > 0,\ l = 1, ..., k,\ e^k = (1, 1, ..., 1),\ y^*_l = \hat{y}_l + \varepsilon,\ \hat{y}_l = \max_{x \in X_0} f_l(x),\ l = 1, ..., k,\ \varepsilon > 0$, and ρ is a positive "sufficiently small" number [2].

On the first glance, the objective function in (2) seems to be difficult to handle. However, observe that optimization problem (2) is equivalent to

$$\begin{gathered} \min s\,, \\ s \geq \lambda_l (y^*_l - f_l(x)) + \rho e^k (y^* - f(x)), \quad l = 1, ..., k\,, \\ x \in X_0\,. \end{gathered} \tag{3}$$

3 Biobjective multidimensional knapsack problems

To solve biobjective multidimensional knapsack problems obtained in the manner described above by a MIP solver, we made use of transformation (3) of optimization problem (2). In case of knapsack problems the resulting problems are all-integer linear problems, thus they can be handled by CPLEX.

The transformation requires $y*$ and ρ. We assumed $\rho = 0.001$. To avoid maximizing both objective functions to to select y^*, we set $y^*_l = \sum_{j=1}^{n} c_{l,j},\ l = 1, 2$, which clearly satisfies the requirement for y^*.

[1] Actually, this solution is properly Pareto optimal (properly efficient), for a formal treatment of this issue cf., e.g., Kaliszewski 2006, Kaliszewski et al. 2016, Ehrgott 2005, Miettinen 1999.

[2] ibidem.

The transformation is parametrized by λ. To simplify modifications of the input LP files we introduced two variables $f1$ and $f2$ equal respectively to the first and the second objective function,

$$-f_1 + \sum_{j=1}^{N} c_{1,j} x_j = 0\,,$$

$$-f_2 + \sum_{j=1}^{N} c_{2,j} x_j = 0\,.$$

Both variables f_1 and f_2 are clearly nonnegative.

The transformed problems have the following form:

$$\min s\,,$$

$$\text{subject to} \quad \begin{cases} s + (\lambda_1 + \rho) f_1 + \rho f_2 \geq \lambda_1 y_1^* + \rho(y_1^* + y_2^*), \\ s + \rho f_1 + (\lambda_2 + \rho) f_2 \geq \lambda_2 y_2^* + \rho(y_1^* + y_2^*), \\ -f_1 + \sum_{j=1}^{N} c_{1,j} x_j = 0\,, \\ -f_2 + \sum_{j=1}^{N} c_{2,j} x_j = 0\,, \\ \sum_{i,j}^{n} a_{i,j} x_j \leq b_i,\ i = 1, ..., m, \\ x_j \geq 0\,. \end{cases} \tag{4}$$

Data for all the problems are organized according to the LP format.

We made the transformed problems publicly available via the web [3]. For each problem λ_1 and λ_2 were set to 0.5 and for different settings of λ_l the first two constraints in (4) have to be modified accordingly.

References

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