# Bipolar queries using various interpretations of logical connectives

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**Abstract.** In [1, 2] we studied various concepts of bipolar queries (cf. Dubois and Prade [3]). We advocated there the use of a fuzzified version of the original crisp definition introduced by Lacroix and Lavency [4]. However, the fuzzification proposed, as general as it was, leaves open the choice of a representation of logical entities, notably the logical connectives and quantifiers. In the present paper we study the influence of the choice some representations that are popular in fuzzy logic on matching degrees of the tuples and their resulting ordering.

## 1 Introduction

One of the most important dimensions of the querying flexibility is the ease with which a user may express his or her requirements as to the data sought. In many scenarios the easiest way is to use in a query some linguistic descriptions such as a "young employee", "high salary", etc. Fuzzy logic provides effective and efficient means to model such linguistic descriptions within queries (cf., e.g., [5]). Much reserach in this area has resulted in some extensions of such powerful query languages like the relational algebra or SQL.

There is a growing interest in some special forms of queries that are important from a practical point of view. This interest dates back to a seminal work of Lacroix and Lavency [4]. Their proposal has been recently generalized to a new broader approach. Basically, it attempts to distinguish *regular* requirements the data to be retrieved should satisfy and some *preferences* of a more subtle, not always obligatory, nature. Thus the name proposed by Chomicki [6] is *queries with preferences*.

The original proposal of Lacroix and Lavency was quickly adopted for the fuzzy case by Bosc and Pivert [7, 8]. In 2002 Dubois and Prade [3] introduced an equivalent concept of a *bipolar query* which provides for another interesting interpretation. Thus even if bipolar queries turn out to be a special case of queries with preferences they still are worthwhile to be distinguished due to their clear semantics that is interesting and relevant from a practical point of view.

Lacroix and Lavency's approach deals with crisp conditions only. We proposed their direct "fuzzification" in [1]. In this paper we further analyze this proposal taking into account various fuzzy logic based interpretations of logical connectives and quantifiers. In Section 2 we gathered all necessary definitions of a wide array of operators most often used to model logical connectives and quantifiers in fuzzy logic. Section 3 recalls briefly the basics of the concept of bipolar queries. Section 4 constitutes the main contribution of this paper and lists the properties of bipolar queries under selected combinations of the logical connective representations. Finally Section 5 summarizes the main results and points out the directions of a further research.

## 2 Preliminaries

The fuzzification of the crisp concept of bipolar queries (cf. Section 3) calls for some interpretation of logical connectives and quantifiers. We will adopt $t$-norms, $t$-conorms, as well as the negation and implication operators, as is usually done.

The $t$-norm operator $t$ is used to represent the fuzzy logical connective of conjunction and is defined as a function $t : [0,1] \times [0,1] \longrightarrow [0,1]$ such that:

$$t(x,1) = x \qquad \forall x \tag{1}$$

$$x \leq y \Rightarrow t(x,z) \leq t(y,z) \qquad \forall x,y,z \tag{2}$$

$$t(x,y) = t(y,x) \qquad \forall x,y \tag{3}$$

$$t(x,t(y,z)) = t(t(x,y),z) \qquad \forall x,y,z \tag{4}$$

In particular we will consider the following popular $t$-norm operators:

$$\textit{minimum} \qquad t_{min}(x,y) = x \wedge y = \min(x,y) \tag{5}$$

$$\textit{product} \qquad t_{\Pi}(x,y) = x \cdot y \tag{6}$$

$$\textit{Lukasiewicz t-norm} \qquad t_W(x,y) = \max(0, x+y-1) \tag{7}$$

The $t$-conorm operator $s$ is used to represent the fuzzy logical connective of disjunction and is defined as a function $s : [0,1] \times [0,1] \longrightarrow [0,1]$ such that:

$$s(x,0) = x \qquad \forall x \tag{8}$$

$$x \leq y \Rightarrow s(x,z) \leq s(y,z) \qquad \forall x,y,z \tag{9}$$

$$s(x,y) = s(y,x) \qquad \forall x,y \tag{10}$$

$$s(x,s(y,z)) = s(s(x,y),z) \qquad \forall x,y,z \tag{11}$$

The properties (8) and (9) imply one more useful property of all $t$-conorms:

$$s(x,1) = 1 \qquad \forall x \tag{12}$$

In particular we will consider the following popular $t$-conorm operators:

$$\textit{maximum} \qquad s_{max}(x,y) = x \vee y = \max(x,y) \tag{13}$$

$$\textit{probabilistic sum} \qquad s_{\Pi}(x,y) = x + y - x \cdot y \tag{14}$$

$$\textit{Lukasiewicz t-conorm} \qquad s_W(x,y) = \min(1, x+y) \tag{15}$$

The negation operator $n$ is used to represent the fuzzy logical connective of negation and is defined as a function $n : [0, 1] \longrightarrow [0, 1]$ such that:

$$n(0) = 1 \tag{16}$$

$$n(1) = 0 \tag{17}$$

$$x \leq y \Rightarrow n(x) \geq n(y) \qquad \forall x, y \tag{18}$$

We will use the most popular negation operator:

$$N(x) = 1 - x \qquad \forall x \tag{19}$$

which additionally possesses the property of *involution*:

$$N(N(x)) = 1 \qquad \forall x \tag{20}$$

Due to their associativity (c.f., (4) and (11)) both $t$-norms and $t$-conorms are naturally generalized to $m$-ary operators, i.e., instead of, e.g., $t(x, t(y, z))$ we will write $t(x, y, z)$.

We will consider combinations of the $t$-norms and $t$-conorms that together with the negation operator $N$ (19) form a De Morgan Triplet, i.e., such a triple $(t, s, n)$ of $t$-norm $t$, $t$-conorm $s$ and negation $n$ that:

$$n(s(x, y)) = t(n(x), n(y)) \tag{21}$$

Due to the involution property of the negation operator $N$ the formula (21) is equivalent to

$$s(x, y) = n(t(n(x), n(y))) \tag{22}$$

The triples $(t_{min}, s_{max}, N)$, $(t_{\Pi}, s_{\Pi}, N)$ and $(t_W, s_W, N)$ are the De Morgan Triplets and will be used in what follows.

The formulae we will use to describe the fuzzy set of tuples being an answer to a bipolar query employ the classical quantifiers. Usually they are modelled in fuzzy logic using the inf and sup operators for the general and existential quantifier, respectively. Thus, the truth values of formulae with these classical quantifiers are computed as follows:

$$\text{truth}(\forall x A(x)) = \inf_x \mu_A(x) \tag{23}$$

$$\text{truth}(\exists x A(x)) = \sup_x \mu_A(x) \tag{24}$$

where $A$ denotes, for simplicity, both a fuzzy predicate symbol and a fuzzy set being its interpretation. The min and max operators may be used instead of inf and sup, respectively, for the finite universe of discourse that will be the case later on.

However, a more consistent treatment of the classical quantifiers in the framework of fuzzy logic may be pursued with the use of $t$-quantifiers and $s$-quantifiers (cf., e.g., [9]). The idea of such a type of quantifiers is based on the observation that a formula with the general (existential) quantifier may be identified with a – possibly infinite – conjunction (disjunction):

$$\forall x A(x) \Leftrightarrow A(a_1) \wedge A(a_2) \wedge \ldots \tag{25}$$
$$\exists x A(x) \Leftrightarrow A(a_1) \vee A(a_2) \vee \ldots \tag{26}$$

where $a_i$ are constants corresponding to the elements of the universe of discourse. In fact we will be interested only in the finite domains. Thus the use of a $t$-norm $t$ ($t$-conorm $s$) to model the conjunction (disjunction) formulae (25)-(26) implies:

$$\text{truth}(\forall x A(x)) = t(\mu_A(a_1), \mu_A(a_2), \ldots, \mu_A(a_m)) \tag{27}$$
$$\text{truth}(\exists x A(x)) = s(\mu_A(a_1), \mu_A(a_2), \ldots, \mu_A(a_m)) \tag{28}$$

In particular for the $t$-norms (5)-(7) and $t$-conorms (13)-(15) one obtains:

$$\forall_{min} x\ A(x) = \min(x_1, \ldots, x_n) \tag{29}$$
$$\forall_{\Pi} x\ A(x) = \prod_{i=1}^{n} x_i \tag{30}$$
$$\forall_{W} x\ A(x) = \max(\sum_{i=1}^{n} x_i - n + 1, 0) \tag{31}$$
$$\exists_{max} x\ A(x) = \max(x_1, \ldots, x_n) \tag{32}$$
$$\exists_{\Pi} x\ A(x) = \sum_{i} x_i - \sum_{i<j} x_i \cdot x_j + \sum_{i<j<k} \prod_{l \in \{i,j,k\}} x_l - \ldots + \tag{33}$$
$$+ (-1)^{n+1} \prod_{1 \le l \le n} x_l \tag{34}$$
$$\exists_{W} x\ A(x) = \min(\sum_{i} x_i, 1) \tag{35}$$

where $x \in X$ and $X = \{x_1, \ldots, x_n\}$. In this paper we use the symbols $\forall$ and $\exists$ with subscripts to denote both the logical symbols of the quantifiers in various formulae and the operators defined by the above equations.

The logical connective of implication is represented by an implication operator which is assumed to be a function $i : [0,1] \longrightarrow [0,1]$ such that:

$$x \le u \Rightarrow i(x,y) \ge i(u,y) \qquad \forall x, y, u \tag{36}$$
$$y \le z \Rightarrow i(x,y) \le i(x,z) \qquad \forall x, y, z \tag{37}$$
$$i(0,y) = 1 \qquad \forall y \tag{38}$$
$$i(x,1) = 1 \qquad \forall x \tag{39}$$
$$i(1,0) = 0 \tag{40}$$

We will consider two most popular approaches to defining this operator with respect to the assumed De Morgan Triplet $(t, s, n)$. These are the so-called $S$-implications and $R$-implications defined as follows:

$$R-\text{implication:} \quad x \rightarrow y = \sup\{z : t(x,z) \le y\} \tag{41}$$
$$S-\text{implication:} \quad x \rightarrow y = s(n(x), y) \tag{42}$$

For both the above types of implications another property holds:

$$i(1, x) = x \qquad \forall x \tag{43}$$

In particular we will consider the following $R$-implication operators:

$$\textit{theGödel implication} \qquad i_{R-min}(x,y) = \begin{cases} 1 \text{ dla } x \leq y \\ y \text{ for } x > y \end{cases} \tag{44}$$

$$\textit{theGoguen implication} \qquad i_{R-\Pi}(x,y) = \begin{cases} 1 & \text{for } x = 0 \\ \min\{1, \frac{y}{x}\} & \text{for } x \neq 0 \end{cases} \tag{45}$$

$$\textit{theLukasiewicz implication} \qquad i_{R-W}(x,y) = \min(1 - x + y, 1) \tag{46}$$

corresponding to the $t_{min}, t_{\Pi}$ and $t_W$ $t$-norms, respectively; and the following $S$-implication operators:

$$\textit{theKleene–Dienes implication} \qquad i_{S-max}(x,y) = \max(1 - x, y) \tag{47}$$

$$\textit{theReichenbach implication} \qquad i_{S-\Pi}(x,y) = 1 - x + x \cdot y \tag{48}$$

corresponding to the $s_{max}$ and $s_{\Pi}$ $t$-conorms, respectively. The $S$-implication operator $i_{S-W}$ is identical with $i_{R-W}$.

## 3 Bipolar queries

The concept of a bipolar query has been introduced by Dubois and Prade [3] in 2002 (the roots of this concept may be traced back to earlier works of Dubois and Prade as well as other authors, cf., e.g., [10]). The idea is to distinguish in a query two types of conditions: *required* and *preferred*. The former have to be unconditionally met by a tuple, while the latter are to some extent optional and less important. However, the facultative character of the latter is rather subtle and cannot be directly grasped with, e.g., the notion of importance weights. A query of this type may be exemplified with:

$$\textit{Find a cheap house preferably near a railway station} \tag{49}$$

Thus, the required, strict condition concerns the price and the preferred condition refers to the distance to a railway station. According to our fuzzification of the Lacroix and Lavency's approach this query is to be interpreted as follows. A house sought has to be *cheap* and *if possible* also *near* the station. The possibility of satisfying both conditions corresponds to the existence of a house that meets both of them. Thus if such a house exists then only such houses are of interest, i.e., belong to the answer of the bipolar query (49). Otherwise it is enough for a house to be cheap to belong to the answer to the query.

Such an interpretation of bipolar queries may be easily expressed using a logical formula in case the conditions are crisp as was done by Lacroix and Lavency [4]. In case of fuzzy conditions their formula may be adapted as we did it in [1].

We adopt the following notation: $X = \{x_j\}$ is a set of tuples to be queried; $C(\cdot)$ and $P(\cdot)$ are, *fuzzy* in general, predicates corresponding to the *required* and *preferred* conditions, respectively. We will identify these predicates with fuzzy sets and $C(x)$ and $P(x)$ will denote their membership function values. Then the Lacroix and Lavency's interpretation of bipolar queries may be more formally expressed by the following description of the set of tuples sought ([4]):

$$\{x \in X : C(x) \wedge (\exists y(C(y) \wedge P(y)) \longrightarrow P(x))\} \tag{50}$$

In [1, 2] we introduced a specific fuzzy version of (50) denoting the characteristic/membership function of the resulting fuzzy set of tuples as $\gamma(C, P, x, X)$:

$$\gamma(C, P, x, X) = \min(C(x), \max(1 - \max_{y \in X} \min(C(y), P(y)), P(x))) \tag{51}$$

In this formula the $(t_{min}, s_{max}, N)$ De Morgan Triplet is used along with the $i_{S-max}$ (47) implication operator and the existential quantifier $\exists$ modelled via the max operator (cf. (28)).

The characteristic feature of the interpretation represented via (51) is that the value of a matching degree, $\gamma(C, P, x, X)$, for a tuple $x$ depends not only on $x$ but also on the whole set of tuples $X$ (what is appropriately accounted for by the fourth parameter, $X$, of $\gamma$).

## 4 Alternative interpretations

Now we will reinterpret the concepts of bipolar queries using three De Morgan Triplets mentioned earlier and related implication operators.

In Table 1 we have collected various reinterpretations of the formula (50) obtained using the particular De Morgan Triplets, implication operators and quantifiers corresponding to the selected $t$-norm (cf., (32)-(35)). These correspond to the formula (51) we proposed in [1]. The subscripts of $\gamma$ indicate the De Morgan Triplet and implication operator used. For example, $\gamma_{\Pi,S}$ refers to the $(t_\Pi, s_\Pi, N)$ De Morgan Triplet and associated $S$-implication operator (cf. (5)-(7), (13)-(15), (41)-(46)).

Let us compare the particular interpretations of Table 1 on a simple example given in Table 2. Let us first compute the truth of $\exists y\ C(y) \wedge P(y)$ for various combinations of the logical connectives:

$$\exists_{min} y\ t_{min}(C(y), P(y)) = 0.8 \tag{52}$$

$$\exists_{\Pi} y\ t_{\Pi}(C(y), P(y)) = 0.96 \tag{53}$$

$$\exists_{W} y\ t_{W}(C(y), P(y)) = 1.0 \tag{54}$$

**Notational remark.** As the truth value of the formula $\exists y\ C(y) \wedge P(y)$ is fixed for a given set of tuples $X$ and a chosen De Morgan Triplet and will be important for our further analysis, we will denote it for brevity by $\exists CP$.

| | |
|---|---|
| $\gamma_{min,S}$ | $\min(C(x), \max(1 - \max_{y \in X} \min(C(y), P(y)), P(x)))$ |
| $\gamma_{min,R}$ | $\begin{cases} C(x) & \text{if } \max_y \min(C(y), P(y)) \leq P(x) \\ \min(C(x), P(x)) & \text{otherwise} \end{cases}$ |
| $\gamma_{\Pi,S}$ | $C(x) \cdot (1 - \exists_{\Pi}(C(y_i) \cdot P(y_i)) \cdot (1 - P(x)) =$ <br> $C(x) \cdot (\prod_i (1 - C(y_i) \cdot P(y_i)) \cdot (1 - P(x)) + P(x))$ |
| $\gamma_{\Pi,R}$ | $\begin{cases} C(x) & \text{if } \exists_{\Pi}(C(y_i) \cdot P(y_i)) = 0 \\ C(x) \cdot \min(\frac{P(x)}{\exists_{\Pi}(C(y_i) \cdot P(y_i))}, 1) & \text{otherwise} \end{cases}$ |
| $\gamma_W$ | $t_W(C(x), i_W(\exists_W\ t_W(C(y), P(y)), P(x)))$ |

**Table 1.** Various reinterpretations of the concept of the bipolar query

| No | $C$ | $P$ | I | | II | | III | | IV | | V | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | *rank* | $\gamma_{min,S}$ | *rank* | $\gamma_{min,R}$ | *rank* | $\gamma_{\Pi,S}$ | *rank* | $\gamma_{\Pi,R}$ | *rank* | $\gamma_W$ |
| 1 | 0.9 | 0.7 | 2 | 0.7 | 2 | 0.7 | 3 | 0.64 | 3 | 0.63 | 2 | 0.6 |
| 2 | 0.8 | 0.8 | 1 | 0.8 | 1 | 0.8 | 2 | 0.65 | 2 | 0.67 | 2 | 0.6 |
| 3 | 0.7 | 1.0 | 2 | 0.7 | 2 | 0.7 | 1 | 0.7 | 1 | 0.7 | 1 | 0.7 |
| 4 | 1.0 | 0.0 | 4 | 0.2 | 4 | 0.0 | 4 | 0.04 | 4 | 0.0 | 4 | 0.0 |

**Table 2.** A comparison of interpretations: an example

It may be easily noticed that various interpretations lead to different matching degrees of particular tuples (i.e., values of the $\gamma$ functions) as well as to different resulting orderings of the tuples. In what follows we study the differences between particular interpretations. We show some properties and examples. In particular we study the effects of choosing between:

- the standard interpretation (i.e., via the max operator) and $t$-norm based interpretation of the classical quantifier $\exists$
- the $R$- and $S$-implications,

**Property 1** For any combination of a $t$-norm, $t$-conorm and $S$-implication or $R$-implication if there exists a tuple $x$ such that $C(x) = 1$ and $P(x) = 1$ then the formula (50) defining a bipolar query turns into $C(x) \wedge P(x)$, where $\wedge$ is represented by a given $t$-norm.

$\diamond$

This property stems from the general characteristics of $t$-norms (1), $t$-conorms (12) and $S$- and $R$-implications (43). This proves that the use of all these operators preserves a basic feature of a bipolar query valid in the crisp case: if there is a tuple satisfying both the required and preferred conditions, then interesting are only tuples satisfying both of them, i.e., the condition (50) turns into a simple conjunction.

**Property 2** For any combination of a $t$-norm, $t$-conorm and $S$-implication or $R$-implication and any set of tuples $X$ if for a tuple $x \in X$ $P(x) = 1$, then the formula (50) turns into $C(x)$.

◇

This property stems from the characteristic feature of the implication operator (39). It is a fairly intuitive property: if a tuple fully satisfies the preferred condition $P$, then its overall matching degree is equal to its satisfaction of the condition $C$. On the other hand, a rank of such a tuple depends also on the matching degrees of other tuples of the set $X$.

**Property 3** The use of the usual fuzzy existential quantifier $\exists_{max}$, i.e., the max operator, instead of $\exists_{\Pi}$ or $\exists_W$:

A. yields greater or equal matching degrees,
B. may change the resulting ordering of the tuples.

◇

Property 3.A stems from the fact that max is the smallest of all $t$-conorms (i.e., from (8) and (9)) and from the monotonicity of any implication operator (36) and $t$-norm operator (2). Property 3.B is illustrated by the following examples. We will denote the matching degrees computed with the use of particular existential quantifiers as $\gamma^{\exists_{\max}}(\ldots)$, $\gamma^{\exists_{\Pi}}(\ldots)$ and $\gamma^{\exists_W}(\ldots)$ with appropriate subscripts and arguments.

**Order reversal by changing the $\exists_{\Pi}$ quantifier to the $\exists_{max}$ quantifier under the De Morgan Triplet $(t_{\Pi}, s_{\Pi}, N)$.** Let us consider two tuples $x, y \in X$ such that $C(x) = 0.6, P(x) = 0.82$ and $C(y) = 0.85, P(y) = 0.4$. Then, $\gamma^{\exists_{\Pi}}_{\Pi,S}(C, P, x, X) = 0.528$, $\gamma^{\exists_{\Pi}}_{min,S}(C, P, y, X) = 0.3$, $\gamma^{\exists_{max}}_{min,S}(C, P, x, X) = 0.535$, $\gamma^{\exists_{max}}_{min,S}(C, P, y, X) = 0.544$. Thus

$$\gamma^{\exists_{\Pi}}_{min,S}(C, P, x, X) \geq \gamma^{\exists_{\Pi}}_{min,S}(C, P, y, X)$$

but

$$\gamma^{\exists_{max}}_{min,S}(C, P, x, X) \leq \gamma^{\exists_{max}}_{min,S}(C, P, y, X)$$

◇

**Order reversal by changing the $\exists_W$ quantifier to the $\exists_{max}$ quantifier under the De Morgan Triplet** $(t_W, s_W, N)$**.** Let us consider two tuples $x, y \in X$ such that $C(x) = 1.0, P(x) = 0.5$ and $C(y) = 0.8, P(y) = 0.8$. Then, $\gamma_W^{\exists_W}(C, P, x, X) = 0.5$, $\gamma_W^{\exists_W}(C, P, y, X) = 0.6$, $\gamma_W^{\exists_{max}}(C, P, x, X) = 0.9$, $\gamma_W^{\exists_{max}}(C, P, y, X) = 0.8$. Thus

$$\gamma_W^{\exists_W}(C, P, x, X) \leq \gamma_W^{\exists_W}(C, P, y, X)$$

but

$$\gamma_W^{\exists_{\max}}(C, P, x, X) \geq \gamma_W^{\exists_{max}}(C, P, y, X)$$

◇

Now let us check what is the effect of changing an $S$-implication by an $R$-implication in (50) while keeping the representation of all other elements fixed.

**Order reversal by changing the $S$-implication to the $R$-implication under the De Morgan Triplet** $(t_{min}, s_{max}, N)$**.** Let us assume that $\exists CP = 0.5$ and let us consider two tuples $x, y \in X$ such that $C(x) = 0.4, P(x) = 0$ and $C(y) = 0.3, P(y) = 0.2$. Then, $\gamma_{min,S}(C, P, x, X) = 0.4$, $\gamma_{min,S}(C, P, y, X) = 0.3$, $\gamma_{min,R}(C, P, x, X) = 0.0$, $\gamma_{min,R}(C, P, y, X) = 0.2$. Thus

$$\gamma_{min,S}(C, P, x, X) \geq \gamma_{min,S}(C, P, y, X)$$

but

$$\gamma_{min,R}(C, P, x, X) \leq \gamma_{min,R}(C, P, y, X)$$

The order reversal exemplified above makes the choice between the implication operators an important issue.

A further analysis leads to the following observation.

**Property 4** Assuming the $(t_{min}, s_{max}, N)$ De Morgan Triplet, for tuples $x$ verifying the conditions:

$$(P(x) \geq \exists CP) \text{ or } ((P(x) \leq \exists CP) \text{ and } (P(x) \geq 1 - \exists CP)) \tag{55}$$

it holds that

1. $\gamma_{min,R}(C, P, x, X) \geq \gamma_{min,S}(C, P, x, X)$
2. replacing the $R$-implication with $S$-implication or vice-versa preserves the resulting order of the tuples, i.e., for $x, y$ verifying (55) it holds:

$$\gamma_{min,S}(C, P, x, X) \geq \gamma_{min,S}(C, P, y, X) \Leftrightarrow \gamma_{min,R}(C, P, x, X) \geq \gamma_{min,R}(C, P, y, X) \tag{56}$$

The validity of this property may be easily proved as follows. In order to show 1., let us observe that for $x$ such that $P(x) \geq \exists CP$ it holds:

$$\gamma_{min,R}(C, P, x, X) = \min(C(x), 1) = C(x) \tag{57}$$

because $i_{R-min}(\exists CP, P(x)) = 1$ due to the fact that $P(X) \geq \exists CP$ while

$$\gamma_{min,S}(C,P,x,X) = \min(C(x), \max(1 - \exists CP, P(x))) \leq C(x) \\ = \gamma_{min,R}(C,P,x,X)$$

On the other hand, for $x$ such that $P(x) \leq \exists CP$ and $P(x) \geq 1 - \exists CP$ it holds:

$$\gamma_{min,S}(C,P,x,X) = \gamma_{min,R}(C,P,x,X) = \min(C(x), P(x)) \tag{58}$$

so that 1. is trivially verified.

In order to show 2. we will consider 3 cases:

I. both $(P(x) \geq \exists CP)$ and $(P(y) \geq \exists CP)$,
II. both $P(x) \leq \exists CP$ and $P(x) \geq 1 - \exists CP$ as well as $P(y) \leq \exists CP$ and $P(y) \geq 1 - \exists CP$,
III. $P(x) \geq \exists CP$ and $P(y) \leq \exists CP$ and $P(y) \geq 1 - \exists CP$.

*Case I.*
Then, $P(x) \geq \exists CP \geq C(x)$ because $\exists CP = \max_y \min(C(y), P(y))$. Moreover (57) holds.

Now, let us assume that the left hand side of (56) holds. It means that:

$$\min(C(x), \max(1 - \exists CP, P(x))) \geq \min(C(y), \max(1 - \exists CP, P(y)))$$

but because $P(x) \geq C(x)$ and $P(y) \geq C(y)$ thus the above reduces to

$$C(x) \geq C(y)$$

which means that $\gamma_{min,R}(C,P,x,X) \geq \gamma_{min,R}(C,P,y,X)$. The proof in the opposite direction of $\Leftrightarrow$ in (56) is obvious.

*Case II.*
Then, (58) holds and thus (56) is trivially verified.

*Case III.*
Now, due to a similar reasoning as in Case I and due to (58), the left hand side of (56) reduces to:

$$C(x) \geq \min(C(y), P(y))$$

But due to (57) and (58) it is identical to the right hand side of (56).

$\diamond$

Thus for the tuples $x$ satisfying (55) their resulting order does not depend on the choice between the $S$-implication and the $R$-implication. The troublesome tuples may appear both for high and low values of $\exists CP$. However in the former case these are less interesting tuples, i.e., such $x$'s that $P(x)$ is small (more precisely $\leq 1 - \exists CP$) while there are tuples well satisfying both $C$ and $P$ (as $\exists CP$ is high). The latter case is worse and should be taken into account while executing bipolar queries with varying logical connectives interpretations.

## 5 Concluding remarks

We discuss various reinterpretations of previously studied [1, 2] definition of bipolar queries. In particular we analyze the effect of the choice of various interpretations of the existential quantifiers and implication operators. The basic conclusion is that such a choice have to be careful as it changes not only the values of matching degree but also the resulting ordering of the tuples. We show also that in some special cases the formula defining bipolar queries reduces to a simpler form under any considered interpretation of logical connectives (cf. Properties 1 and 2).

A further research is needed to classify effects of other choices of logical connectives interpretations. Moreover we plan to extend the similar analysis to the case of the *winnow* operator we introduced in [2] as well as to its relation with bipolar queries.

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