Towards a General and Unified Characterization of Individual and Collective Choice Functions under Fuzzy and Nonfuzzy Preferences and Majority via the Ordered Weighted Average Operators

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A fuzzy preference relation is a powerful and popular model to represent both individual and group preferences and can be a basis for decision-making models that in general provide as a result a subset of alternatives that can constitute an ultimate solution of a decision problem. To arrive at such a final solution individual and/or group choice rules may be employed. There is a wealth of such rules devised in the context of the classical, crisp preference relations. Originally, most of the popular group decision-making rules were conceived for classical (crisp) preference relations (orderings) and then extended to the traditional fuzzy preference relations. In this paper we pursue the path towards a universal representation of such choice rules that can provide an effective generalization—for the case of fuzzy preference relations—of the classical choice rules. © 2008 Wiley Periodicals, Inc.

1. INTRODUCTION

Because of an increasing complication of virtually all problems faced by modern societies, which involve issues concerned with technological, management, financial, political, human, etc. aspects, the relevance of knowledge is rapidly increasing. Knowledge is, on the one hand, a sparse "commodity" in the sense that it is possessed by some specific members of the society, not everybody. On the other hand, it is distributed in the sense that not all members of an elitist group of the "chosen ones" mentioned above possess knowledge on some topic of relevance or interest to the same degree. Therefore, to most effectively and efficiently employ that knowledge, a (relatively small) subgroup of the above elite should be elected,

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and their individual testimonies should be used (taken into account or aggregated) to arrive at a group opinion. It can be expected that this process arriving at a democratic, agreed upon, opinion, maybe a consensus, by employing in a democratic and synergistic way knowledge and expertise of different people, should lead to better choices and decisions.

One can look at the above problem of arriving at a collectively agreed upon decision from different points of view. In this article we will view it as a group decision-making problem, and we will adopt some choice function related perspective that provides a considerable generality.

The essence of decision making, which is one of the most crucial and omnipresent human activities, is basically to find a best alternative (option, variant, ...) from among some feasible (relevant, available, ...) ones. This general importance has naturally implied that decision making has become a subject of intensive research which has finally reached a "mature" stage of formal, mathematical models that try to formalize the human rational behavior.

Initially, this rationality has been equated with the maximization of some utility (value) function, and this has lead to powerful formal results. Unfortunately, more and more experiments have clearly indicated that the human behavior is rarely consistent with the maximization of a (expected) utility function, and some attempts to "soften" decision-making models to make them more human consistent have been made. The softening concerned a plausible modification of assumptions on, say, the human preferences, axioms underlying the (expected) utility-based approach, etc. Approaches related to fuzzy logic are most relevant for our discussion, and one can cite here, Blin[1] or Blin and Whinston[2] for earlier works, and, Salles[3] for later works.

However, decision making in real world rarely proceeds in such a simple setting as above, and normally one faces both multiple criteria and multiple decision makers. In this paper we will consider the latter situation. Among multiperson decision-making settings, group decision making (which is meant here to be practically equivalent to social choice) is an important class. Its essence, for the purposes of our discussion, is basically as follows. There is a set of alternatives and a set of individuals who provide their testimonies concerning the alternatives. Usually, these testimonies are assumed to be preferences over the set of options, and this is also the case in this paper. The problem is to find a solution, that is, an alternative (or a set of alternatives) which is best acceptable by the group of individuals as a whole. For a different point of departure, involving choice sets or utility functions, we may refer the reader to, for example, Kim,[4] Salles,[3] or Tanino.[5] In both cases, we may view the process as an attempt to effectively and efficiently employ knowledge distributed over a group of (most) qualified experts.

This group decision-making problem is simple and seems an ideal vehicle to arrive at perfect decisions. Unfortunately, it has been shown that some "obvious" properties are not satisfied, and no matter what we do, solutions obtained cannot guarantee to fulfill all such obviously justified conditions. This is mainly reflected by some impossibility theorems, paradoxes, etc. Among them, by far the best known negative result is the so-called Arrow's impossibility theorem (cf., Arrow[6]) that basically states that, under some plausible assumptions, there is no social (welfare) choice function that: has an unrestricted domain, is independent of irrelevant options, satisfies the Pareto condition, satisfies the non dictatorship condition. Another well-known negative result is due to Gibbard and Satterthwaite (cf., Gibbard[7]) that basically states that all (universal and non trivial) social decision functions are either manipulable or dictatorial. Among other negative results, we may note, say, McKelvey's and Schofield's findings on the instability of solutions in spatial contexts, etc. (cf., Nurmi[8,9]). Basically, all these negative results might be summarized as follows: no matter which group choice procedure we employed, it would satisfy one set of plausible conditions but not another set of equally plausible ones. We will not discuss these conditions in detail, and will refer the interested reader to the literature, cf., for example, Nurmi.[8] Unfortunately, this general property pertains to all possible choice procedures, so that attempts to develop new, more sophisticated choice procedures do not seem very promising in this respect. Much more promising seems to be to modify some basic assumptions underlying the group decision-making process. This line of reasoning is also basically assumed here. We will concentrate on approaches based on fuzzy logic. For other approaches, see, for example, Intrilligator[10,11] for a probabilistic approach or references 55–60 for a fuzzy approach.

Since the process of decision making, notably of group type, is centered on the human beings, with their inherent subjectivity, imprecision, and vagueness in the articulation of opinions, fuzzy sets have been used in this field for a long time.

A predominant research direction is here based on the introduction of an individual and social fuzzy preference relation.

Basically, suppose that we have a set of $M \geq 2$ options, $S = \{s_1, \ldots, s_M\}$, and a set of $N \geq 2$ *individuals* (experts) $E = \{e_1, \ldots, e_N\}$. An individual fuzzy preference relation in $S \times S$ of individual $e_k \in E$ assigns a value in the unit interval for the preference of one alternative over another. An immediate extension is that instead of a value, one can assume that some interval, fuzzy number, or—more generally—a linguistic value can be used to express the intensity of preference. The introduction of fuzzy preference relations may alleviate difficulties in social choice, confer Ovchinnikov,[12] Barrett, Pattanaik, and Salles,[13] or Salles.[3]

Basically, to employ fuzzy preference relations to find solutions, two lines of reasoning may be followed here (cf., Kacprzyk[14–16]):

$$\text{– a direct approach} \quad \{R_1, \ldots, R_N\} \to \text{solution} \tag{1}$$

$$\text{– an indirect approach} \quad \{R_1, \ldots, R_N\} \to R \to \text{solution}$$

that is, in the first case we determine a *solution* just on the basis of individual fuzzy preference relations, and in the second case we form first a social fuzzy preference relation R which is then used to find a solution.

The relations between individual and group choice may be depicted as in Figure 1.[17] In Figure 1 we have

$\{R_1, \ldots, R_N\}$	individual preference relations of the experts;
$\{S_1, \ldots, S_N\}$	subsets of "best" options implied by particular individual preference relations (individual choice sets);

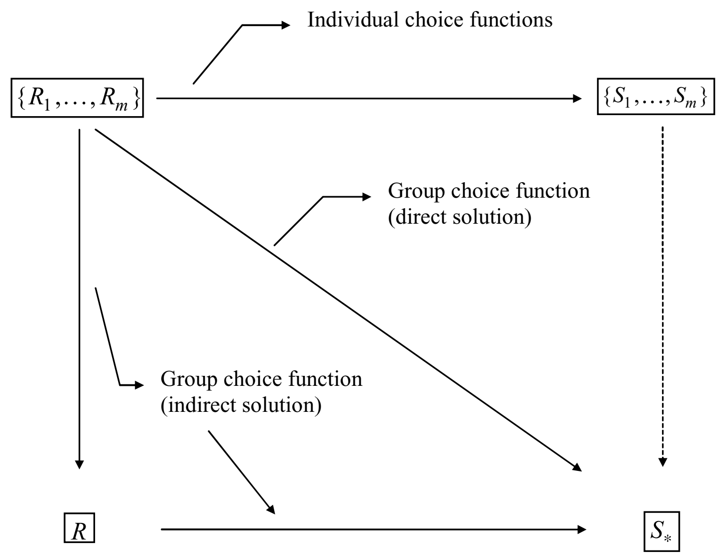

Figure 1. Preferences aggregation.

R group preference relation determined for the set of individual preference relations;
S_* subset of "best" options implied by all individual preference relations taken together (group choice set).

The fourth side of the rectangle, represented by a dashed line, refers to the direct aggregation of the individual choice sets, S_i, to the group choice set. The problem of this type, in the fuzzy case, was studied by many authors (for references, see[17]).

Another basic element underlying group decision making is the concept of a majority—notice that a solution is to be an option (or options) best acceptable by the group as a whole, that is, by (at least!) most of its members since in practically no real nontrivial situation it would be accepted by all. Some of the above mentioned problems, or negative result, with group decision making are closely related to a (too) strict representation of majority (e.g., at least a half, at least 2/3, . . .). A natural line of reasoning is to somehow make that strict concept of majority closer to its real human perception by making it more vague. A good, often cited example in a biological context may be found in Loewer and Laddaga[18]: ". . . It can correctly be said that there is a consensus among biologists that Darwinian natural selection is an important cause of evolution though there is currently no consensus concerning Gould's hypothesis of speciation. This means that there is a widespread agreement among biologists concerning the first matter but disagreement concerning the second . . . ," and it is clear that a rigid majority as, for example, more than 75% would evidently not reflect the essence of the above statement. However, it should be noted that there are naturally situations when a strict majority is necessary, for obvious reasons, as in all political elections.

A natural manifestations of such a "soft" majority are the so-called linguistic quantifiers as, for example, most, almost all, much more than a half, etc. Such linguistic quantifiers can be, fortunately enough, dealt with by fuzzy-logic-based calculi of linguistically quantified statements as proposed by Zadeh[19] and Yager.[20] Moreover, Yager's[21] (cf., Yager and Kacprzyk[22]) ordered weighted averaging (OWA) operators can be used for this purpose.

These calculi have been applied by the authors to introduce a fuzzy majority (represented by a fuzzy linguistic quantifier) into group decision making and consensus formation models (Kacprzyk;[14,15,23–26] Kacprzyk, Fedrizzi, and Nurmi;[27] Kacprzyk and Nurmi;[28,29] Nurmi and Kacprzyk;[30] Nurmi et al.[31]). See also Kacprzyk and Nurmi[29] for a comprehensive review.

Evidently, the group decision making is basically an aggregation process, because individual testimonies are aggregated. On the other hand, it is a choice process because some options should be chosen, that is some solution(s) should be found. In this paper, we will present some unified view of that aggregation and choice process in the sense that we will show that we can use Yager's[21] OWA operators to derive a general expression for a multitude of individual and collective choice functions under fuzzy preferences and a fuzzy majority.

Our points of departure are the following: the families of individual choice rules defined by Barrett et al.[13] and the linguistic aggregation rule as proposed by Kacprzyk[14,16]. In both cases we further develop and generalize an aggregation-oriented attitude, extending our previous papers (Kacprzyk and Zadrozny.[32,33]) It makes possible to propose universal formulas subsuming many well-known individual and group choice concepts and rules. Our main tool is Yager's OWA operators.[21]

2. FUZZY PREFERENCE RELATIONS

We assume the following: a set of options is $S = \{s_1, \ldots, s_M\}$, and a set of individuals (experts) is $E = \{e_1, \ldots, e_N\}$. An individual presents his or her preferences in respect to each pair of options, thus defining a preference relation R, modeled as a fuzzy subset of $S \times S$. It may be identified with its membership function $\mu_R(\cdot,\cdot)$. Assuming a reasonable (small) cardinality of the set S, it is convenient to represent a preference relation R_k of individual e_k as a matrix:

$$[r_{ij}^k] = [\mu_{R_k}(s_i, s_j)], \forall i, j, k \tag{2}$$

Usually, the fuzzy preference relation is assumed to meet certain conditions. If the reciprocity property is assumed, that is:

$$\mu_R(s_i, s_j) + \mu_R(s_j, s_i) = 1, \forall i \neq j$$

then such a relation is referred to as the *fuzzy tournament*; confer Nurmi and Kacprzyk.[30] In such a case the following interpretation is justifiable:

$$\mu_R(s_i, s_j) = \begin{cases} 1 & \text{definite preference of } s_i \text{ over } s_j \\ c \in (0.5, 1) & \text{preference to some extent} \\ 0.5 & \text{indifference} \\ d \in (0, 0.5) & \text{preference to some extent} \\ 0 & \text{definite preference of } s_j \text{ over } s_i \end{cases} \quad (3)$$

A more general approach consists in considering the relation R as a weak preference relation: $R(s_i,s_j)$ means then "s_i is not worse than s_j." According to this semantics, only the reflexivity (sometimes together with the transitivity) of R is postulated: $\mu_R(s_i,s_i) = 1$. Such a relation may conveniently be treated as a composition of three relations: *strict preference* P, *indifference* I, and *incomparability* J. They form together what is called sometimes a *preference structure*: (P, I, J)—confer the book of Fodor and Roubens.[34] In the case of classical crisp preference relations, the following definitions are assumed:

$$P(s_i, s_j) \Leftrightarrow R(s_i, s_j) \wedge \neg R(s_j, s_i) \quad (4)$$

$$I(s_i, s_j) \Leftrightarrow R(s_i, s_j) \wedge R(s_j, s_i) \quad (5)$$

$$J(s_i, s_j) \Leftrightarrow \neg R(s_i, s_j) \wedge \neg R(s_j, s_i) \quad (6)$$

which implies

$$R = P \cup I \quad (7)$$

and the mutual disjointness of the relations P, I, and J.

In the case of fuzzy preference relations, these definitions cannot be directly adopted just by replacing the conjunction and negation symbols with their fuzzy logic counterparts in Equations 4 to 6. To preserve (7) some special operators *po*, *io*, and *jo* have to be defined as membership functions of the relations P, I, and J, respectively. One possible selection of these operators, based on the Łukasiewicz De Morgan triple, is the following:

$$\mu_P(s_i, s_j) = po(s_i, s_j) = \max(\mu_R(s_i, s_j) - \mu_R(s_j, s_i), 0) \quad (8)$$

$$\mu_I(s_i, s_j) = io(s_i, s_j) = \min(\mu_R(s_i, s_j), \mu_R(s_j, s_i)) \quad (9)$$

$$\mu_J(s_i, s_j) = jo(s_i, s_j) = \min(1 - \mu_R(s_i, s_j), 1 - \mu_R(s_j, s_i)) \quad (10)$$

Then (7) is verified if $\cup$ is interpreted using Łukasiewicz disjunction. It may be noted that (8) is a direct interpretation of (4), where $\wedge$ is interpreted via Łukasiewicz t-norm ($x \wedge y = \max(x + y - 1,0)$) while (9)–(10) correspond to (5)–(6), where $\wedge$ is interpreted via the min operator. For the details, including the underlying axioms see the book of Fodor and Roubens.[34]

For the purposes of this paper, we will not use the more comprehensive preference system mentioned above and will only deal with traditional fuzzy preference relations which is sufficient conceptually.

In our discussion of individual and group choice rules we will consider both the general forms of fuzzy preference relations and their special cases such as fuzzy tournaments (3) and *crisp linear orderings*, that is, crisp relations R defined on $S \times S$ and possessing the properties of:

- reflexivity: $\forall s_i \in S \; R(s_i, s_i)$
- antisymmetry: $\forall s_i, s_j \in S \; R(s_i, s_j) \wedge R(s_j, s_i) \rightarrow s_i = s_j$
- transitivity: $\forall s_i, s_j, s_k \in S \; R(s_i, s_j) \wedge R(s_j, s_k) \rightarrow R(s_i, s_k)$

The latter are considered in many classical voting/choice rules. We will show that our general scheme of group choice covers many of these rules when applied to crisp linear orderings.

3. FUZZY MAJORITY AND THE OWA OPERATORS

Fuzzy majority constitutes a natural generalization of the concept of majority in the case of a fuzzy setting within which a group decision-making problem is considered. A fuzzy majority was introduced into group decision making under fuzziness by Kacprzyk[14–16] and then considerably extended in the works of Fedrizzi, Herrera, Herrera-Viedma, Kacprzyk, Nurmi, Verdegay, Zadrożny etc. (see, e.g., a review by Kacprzyk and Nurmi,[29] and papers cited in the bibliography).

Basically, Kacprzyk's[14–16] idea was to equate a fuzzy majority with a fuzzy linguistic quantifiers, which often appear in a natural language discourse. Linguistic (fuzzy) quantifiers exemplified by expressions like “most,” “almost all,” etc. allow for a more flexible quantification than the classic general and existential quantifiers.

There exist a few approaches to the linguistic quantifiers modeling. Basically, we are looking for the truth of a proposition:

Most objects posses a certain property

that may be formally expressed as follows:

$$\underset{x \in X}{\mathrm{Q}} \; P(x) \tag{11}$$

where Q denotes a fuzzy linguistic quantifier (in this case “most”), $X = \{x_1, \ldots x_m\}$ is a set of objects, $P(\cdot)$ corresponds to the property. It is assumed that the property P is fuzzy and its interpretation may be informally equated with a fuzzy set and its membership function, that is:

$$\mathrm{truth}(P(x_i)) = \mu_P(x_i)$$

The first approach, proposed by Zadeh,[19] is called a *calculus of linguistically quantified propositions*. Here, a linguistic quantifier is represented as a fuzzy set $Q \in \mathrm{F}([0,1])$, where $\mathrm{F}(A)$ denotes the family of all fuzzy sets defined on A. For practical reasons its membership function is often assumed piece-wise linear. Thus, the fuzzy set corresponding to the fuzzy quantifier Q ("most") may be defined by, for example, the following membership function:

$$\mu_Q(y) = \begin{cases} 1 & \text{for} \quad y \geq 0.8 \\ 2y - 0.6 & \text{for} \quad 0.3 < y < 0.8 \\ 0 & \text{for} \quad y \leq 0.3 \end{cases} \tag{12}$$

The truth of the proposition (11) is determined:

$$\text{truth}(\text{Q}\, P(X)) = \mu_Q\left(\sum_{i=1}^{m} \mu_P(x_i)/m\right) \tag{13}$$

where $m = \text{card}(X)$.

The linguistic quantifier may be treated as a flexible aggregation operator situated somewhere between the aggregation operators corresponding to the AND and OR logical operators. These logical operators in fuzzy logic are usually interpreted as the min and max operators, respectively. Thus, they aggregate a sequence of values given on input by taking the smallest or largest value of them. In this logical framework they compute the truth value of a conjunction or disjunction of elementary propositions, each true to a degree from [0,1]. A linguistic quantifier, depending on the actual shape of its membership function exemplified by (12), offers aggregation schemes ranging between these two extreme cases of conjunction and disjunction.

Another approach to the modeling of fuzzy linguistic quantifiers (and flexible aggregation) is by using Yager's OWA operators[21] (see also Yager and Kacprzyk's[22] volume).

An interesting and powerful tool may here also be the OWmin and/or OWmax operators, the Sugeno and Choquet integrals, etc. Moreover, a different view of fuzzy quantifiers and their related calculi due to Gloeckner[35] may be employed. These tools will not be used here because—from the point of view of our goal—they all make it possible to provide an aggregation needed so that conceptually they may be viewed the same.

The concept of an OWA operator was introduced by Yager[21] (cf., Yager and Kacprzyk[22]). An OWA operator O of dimension n may be briefly described as follows:

$$O : \Re^n \rightarrow \Re$$
$$W = [w_1, \ldots, w_n], \quad w_i \in [0, 1], \quad \sum_{i=1}^{n} w_i = 1$$
$$O(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \quad b_j \text{ is } j\text{-th largest of the } a_i \tag{14}$$

Here, a_i are values to be aggregated and W is a vector of weights defining the operator. Depending on the assumed vector W these operators provide for a unified representation of such aggregation schemes as the minimum and maximum, median or average. When aggregated values are meant as truth values in fuzzy (multivalued) logic, these schemes directly correspond to various types of quantification. Namely, the classical general (∀) and existential (∃) quantifiers may be interpreted as the min and max, respectively. We denote:

$$O_{\forall} : \forall \rightarrow W = [0, \ldots, 0, 1]$$

$$O_{\exists} : \exists \rightarrow W = [1, 0, \ldots, 0] \quad (15)$$

It is worth noticing that $O_{\forall}$ and $O_{\exists}$ are *dual* in the sense that

$$O_{\forall}(a_1, \ldots, a_n) = 1 - O_{\exists}(1 - a_1, \ldots, 1 - a_n) \quad (16)$$

More precisely, every two OWA operators such that $w_i^1 = w_{n-i+1}^2$, where w_k^p is the kth weight of pth operator, are dual.

The following weight vectors define other OWA operators useful for our purposes:

O_A : ***average*** $W = [1/n, \ldots, 1/n]$

O_{most} : ***most*** (17)

O_{maj} : ***classical crisp majority***

$$W = [0, \ldots, 0, 1, 0, \ldots, 0] \; w_{(n/2)+1} \; or \; w_{(n+1)/2} = 1$$

The O_A yields the usual arithmetic mean but when applied to arguments from {0,1}, it may be regarded as counting the number of 1s in the argument. If the argument is treated as the characteristic function of some set, then O_A may be used to compare the cardinalities of such sets—the higher the value of O_A the higher cardinality of the corresponding set. Obviously, the result of aggregation using O_A is also proportional to the sum of its arguments. This will be useful for our purposes.

Ordered weighted averaging operators possess the following properties relevant for our considerations:[21,36]

- neutrality: $O(a_1, \ldots, a_n) = O(a_{i1}, \ldots, a_{in})$ for all $(a_1, \ldots, a_n) \in \mathrm{R}^n$, where $i1, \ldots, in$ is a permutation of $1, \ldots, n$
- monotonicity: $a_i < b_i \Rightarrow O(a_1, \ldots, a_i, \ldots, a_n) \leq O(a_1, \ldots, b_i, \ldots, a_n)$,
- idempotence: $O(a, a, \ldots, a) = a$,
- compensativeness: $\min_i a_i \leq O(a_1, \ldots, a_n) \leq \max_i a_i$,

Yager and Filev[37,38] proposed a new class of OWA like operators called IOWA—*Induced OWA*. They share the modus operandi with the original OWA operators but accept two vectors as input. The first vector plays the same role as the vector $(a_1, \ldots, a_n)$ in the case of OWA operators—see (14). This vector is sorted non increasingly as in (14) but according to the corresponding values of the second input vector instead of a_i's themselves. The IOWA operators seem to be gaining popularity in the (fuzzy) decision theory literature, for instance, Pasi and Yager[39] proposed an IOWA-based operator in order to determine a value representative for a *majority* of elements a_i. This is very relevant for our considerations here. It may be employed, for example, to derive fuzzy group preference relation, as assumed in the indirect approach (1).

4. INDIVIDUAL CHOICE UNDER FUZZY PREFERENCES

Usually, the preference formalization is just an intermediate step, preceding the selection of one or more options meant as a best solution. Sometimes this selection is fairly obvious and trivial step. For example, if a preference relation is an ordering, it is reasonable to assume the first option (maybe accompanied by the second, third, etc.) as the best option(s). Some other rules may be needed if one does not impose strong enough requirements as to the properties of the preference relation. Such rules are called *choice rules* (*functions*) and, in the crisp context, are formally described as follows:

$$C(X, R) = X_0 \quad X_0 \subseteq X, \quad \forall X \subseteq S \tag{18}$$

Thus, the choice rule defined for the preference relation R by the function C have to be described for all subsets of the set of options S. For each X, it produces its subset meant as the *choice set* of the best options in X with respect to R. The notion of the choice function may also be meant in a more general sense. Namely, an individual may be assumed to express his or her opinion directly as a choice function instead of a preference relation. Then, a choice function is defined as $C{:}2^S \to 2^S$, such that $C(X) \subseteq X$. This line of research is pursued in reference 40. In this paper, while discussing the individual choice function we always refer to the definition (18).

Particular choice functions refer to some *rationality concepts* sanctioning this or other selection of options. A classic concept of a choice function is based on *greatestness*,[41] that is:

$$C_1(X, R) = \{s_i \in X : \forall_{\substack{s_j \in X \\ s_i \neq s_j}} s_i R s_j\} \tag{19}$$

Hence, the set of chosen (preferred) options, $C_1(X,R)$, consists of options greater than all other options in X, in the sense of R. The problem is that unless we assume some relatively strong properties of R, the set of the greatest elements may easily be empty. Thus, often a similar notion of *non dominance* is employed resulting in the following definition of the choice function:

$$C_2(X, R) = \{s_i \in X : \neg\exists_{\substack{s_j \in X \\ s_i \neq s_j}} s_j R s_i\} \tag{20}$$

Hence, in this case the set of chosen options, $C_2(X,R)$, consists of the options being non dominated, in the sense of R, by any other option in X. Often another version of (20) is employed where the relation R on the right-hand side is replaced with its strict preference component P, confer (4). The chances for a non emptiness of C_2 are higher but still not high enough to solve the choice problem.[42] If the antisymmetry and completeness of relation R is assumed, both C_1 and C_2 are identical.

Both the previous definitions of a choice function have an elementwise character, that is, they describe what conditions an option has to meet to be qualified as a member of the choice set. Another possibility is to use a rationality concept that directly applies to a set of options. Aizerman and Aleskerov[40] call such definitions of choice functions as *integral*. Some of them are discussed by Schwartz[43] and Kitainik.[42] Here we will refer to two of them, based on the so-called GETCHA and GOCHA[43] concepts, that form generalizations of C_1 and C_2, respectively. However, contrary to their element-wise counterparts, they always produce a non empty choice set.

A GETCHA-based choice function, applicable for antisymmetric relations R, may be defined as follows:

$$C_3(X, R) = Y \quad \text{if} \quad \underset{s_j \in \bar{Y}}{\forall} \; \underset{s_j \in Y}{\forall} \; s_i R s_j \quad Y \subseteq X, \quad \bar{Y} = X \backslash Y \tag{21}$$

and no proper subset of Y possesses this property.

A GOCHA-based choice function is defined as follows

$$C_4(X, R) = \bigcup_l Y_l \quad \text{if} \quad \underset{s_i \in Y_l}{\forall} \; \underset{s_j \in \bar{Y}_i}{\neg\exists} \; s_i R s_j \quad Y_l \subseteq X, \quad \bar{Y}_l = X \backslash Y_l \tag{22}$$

Again, as in case of C_1 and C_2, if the properties of antisymmetry and completeness are satisfied for R, then both the functions C_3 and C_4 are identical.

To summarize, for crisp antisymmetric and complete preference relation the GETCHA (or GOCHA) may be treated as a representative for all four individual choice functions mentioned because under the assumed properties of the preference relation they are identical and, moreover, they generalize the corresponding element-wise definitions.

The investigation of individual choice functions under fuzzy preference relation also started with the formula (19). However, in the fuzzy case it is not obvious how xRy should be interpreted. Nurmi[44] assumes the interpretation of fuzzy preference relations given by (3) and defines the *consensus winner* and *α-consensus winner* choice sets as follows, respectively:

$$C_5(X, R) = \{s_i \in X : \underset{i \neq j}{\forall} \; r_{ij} > 0.5\} \quad \text{and}$$

$$C_6(X, R) = \{s_i \in X : \underset{i \neq j}{\forall} \; r_{ij} \geq \alpha > 0.5\}.$$

where $r_{ij} = \mu_R(s_i, s_j)$, cf. Section 2.

These choice functions are a proper generalization of (19): if $r_{ij} \in \{0, 1\}$, then both are identical with (19) and, moreover, even for $r_{ij} \in [0, 1]$ the choice set is either of a single element or empty. In fact an α-cut of the relation R is used and (19) is directly employed to define these concepts.

Nurmi[44] introduced also a counterpart of (20), the *minimax consensus winner*, defined as

$$C_7(X, R) = \{s_i : i = \arg \min_j \max_k r_{kj}\} \tag{23}$$

Again, for $r_{ij} \in \{0, 1\}$ C_7 behaves like C_2 (except when C_2 is empty—then C_7 is equal X.) Therefore, for crisp counterparts of the preference relations considered here, both the C_6 and the C_7 are identical. Note that these choice functions produce crisp choice sets.

To obtain non empty choice sets for a wider class of fuzzy preference relations, Kacprzyk[14–16] introduced the concept of a Q-consensus winner defined, informally, as follows:

$$C_8(X, R) = \{s_i \in X : \underset{i \neq j}{Q} (r_{ij} > \alpha)\} \tag{24}$$

where Q is a linguistic quantifiers in the sense of Zadeh.[19] The idea is to use a quantifier referring to the concept of a *fuzzy majority*, thus to choose options that are preferred over, for example, *most* of the other options.

Some other approaches, also intended for other types of fuzzy preference relations, include the work of Świtalski,[45] Roubens,[46] Kitainik,[42] Barrett et al.,[13] Orlovski,[47] and others.[12,48–50]

Seeking a generalization of the above approaches we refer to Barrett et al.[13] They start with an observation related to the nature of known individual choice functions. Namely, most of them assess the membership of a given option, s_i, in the choice set employing a kind of aggregation of the preferences of all other options against option s_i. It may be formally stated as:

$$\mu_{C(X,R)}(s_i) = \text{AGG}\{r_{ij}\}_{i \neq j} \tag{25}$$

or

$$\mu_{C(X,R)}(s_i) = \text{AGG}\{r_{ji}\}_{i \neq j} \tag{26}$$

where the left-hand side denotes the membership degree of a given option in the (fuzzy) choice set and the symbol AGG in the right side stands for some aggregation operator. Hence, they study the whole family of individual choice functions using either (25) or (26) with such aggregation operators as the max, min, and summation. They consider also other variations of (25) and (26), but we will concentrate on those mentioned earlier.

Before showing how the OWA operators can be applied to represent the whole family of the individual choice functions, a short remark is needed. The functions discussed in Barrett et al.[13] produce a fuzzy choice set that is obvious since they deal with fuzzy preference relations. On the other hand, usually the final choice should be crisp and we have to decide how to "defuzzify" the result obtained. Moreover, we argue that our OWA-based definitions of individual choice functions agree with original, classic versions, whereas some of these OWA-based definitions produce fuzzy results even for crisp preferences. Then, our claim of correspondence between the proposed and the classical versions refers to the "defuzzified" result. Thus, we need some defuzzification procedure. We will use the most obvious one, employed also in Barrett et al.,[13] namely, for the choice functions evolving from (25) and (26) we use the max and min operators, respectively.

Let us now proceed with some definitions of individual choice functions by using the OWA operator. Obviously, C_1 defined by (19) may be expressed as follows:

$$\mu_{C_1}(s_i) = O_{\forall} R(s_i, s_j) \quad \text{(max-MF)} \tag{27}$$

Here, and in what follows, the right-hand side should be read as $O_{\forall}(r_{i1}, r_{i2}, \ldots, r_{iM})$, that is an index used on the left-hand side (here: i) is fixed on the right-hand side and the aggregation is done over the second index used on the right-hand side (here: j). In parenthesis we quote the label assigned to the function in Barrett et al.[13]—here it is max-MF.

Assuming the negation represented by neg$(x) = 1\text{-}x$, C_2 may be expressed as: $\mu_{C_2}(s_i) = 1 - O_{\exists} R(s_j, s_i)$. Using the duality defined by (16) and the reciprocity, this may be transformed to:

$$\mu_{C_2}(s_i) = 1 - O_{\exists} R(s_j, s_i) = O_{\forall}(1 - R(s_j, s_i)) = O_{\forall}(R(s_i, s_j))$$

confirming the identity of both choice functions for antisymmetric and complete preference relations, as we noted earlier. Obviously, defuzzification of the choice will be done using the max operator.

The C_7 (23) may be expressed as

$$\mu_{C_7}(s_i) = O_{\exists} R(s_j, s_i) \quad \text{(min-MA)} \tag{28}$$

but, this time the defuzzification of the choice set goes through the min operator.

The above examples introduce a general scheme for individual choice functions *ICR*:

$$\mu_{ICR}(s_i) = O(a_1, \ldots, a_M) \tag{29}$$

$$a_j = \mu_R(s_i, s_j) \quad \text{or} \tag{30}$$

$$a_j = \mu_R(s_j, s_i) \tag{31}$$

Thus, various choice rules can be obtained by the selection of the OWA operator O in (29), the use of the (30) or (31) and related to that defuzzification via max or min operator, respectively. Besides the examples of this scheme discussed above also some other may be found interesting. For example, using the O_{avg} in (29) along with

(30) and max operator one obtains the rule choosing an option dominating highest *number* of other options. In case of a crisp preference relation R, the number is meant as the cardinality of the crisp set of dominated options whereas in case of a fuzzy preference relation R the number is meant as the crisp cardinality (the so-called sigma-count, ΣCount) of the fuzzy set of dominated options. It is worth noticing that such a rule applied to a crisp linear ordering picks up the first element of the ordering as the only element of the choice set. This will be useful for the description and analysis of some collective choice rules (CCRs) discussed later on in this article.

The most general form of individual choice functions we considered, that is, the GETCHA (or GOCHA), is difficult to describe using the proposed scheme involving the OWA operator. The scheme has an element-wise character, while the GETCHA concept refers to the integral definition. It may be observed that a subset of the choice set produced by C_3 function, that is, the GETCHA, is obtained when using the following function:

$$\mu_{C_3^*}(s_i) = O_A R(s_i, s_j) \quad \text{(max-SF)} \tag{32}$$

More precisely, it may be proved that for crisp antisymmetric and complete preference relations given above the formula coupled with the max operator-based defuzzification yields the subset of the choice set determined by the original definition of GETCHA (21), that is, $C_3^*(X, R) = \{s_i \in X : i = \arg\max O_A R(s_i, s_j)\}$ is the subset of $C_3(X,R)$. A simple proof follows. Let us assume $s \in C_3^*$ and $s \notin C_3$. Then, there exists $s' \in C_3$ such that $R(s', s)$ and $\forall_j R(s, s_j) \Rightarrow R(s', s_j)$. That is because the set C_3 is always non empty and if s does not belong to it, neither do all s_j such that $R(s,s_j)$. Hence, $O_A R(s',s_j) > O_A R(s,s_j)$ which contradicts $s \in C_3^*$. This completes the proof.

Obviously, the integral definitions may be also expressed using the OWA operators as these can directly represent the classical quantifiers. Thus, for example, the GETCHA may be presented as:

$$\mu_{GETCHA}(Y) = O_\forall^i O_\forall^j R(s_i, s_j) \quad i \in \{k : s_k \in Y\} \text{ and } j \in \{k : s_k \notin Y\} \tag{33}$$

Thus, we effectively define a fuzzy set in the space 2^Y. Moreover, by assigning the values of importance to elements of the set Y, by using (33) we can assess how particular fuzzy sets Y meet conditions of being the GETCHA choice set.

5. COLLECTIVE CHOICE UNDER FUZZY PREFERENCES

A CCR describes how to determine a set of preferred options starting from the set of individual preference relations. Thus, it may be informally represented as follows:

$$\{R_1, \ldots, R_N\} \to 2^N$$

Note that this expression reflects the direct approach to the determination of a solution. In fact, for our discussion later on it is not important whether we assume the collective choice function to be derived directly as above, that is, via $\{R_1, \ldots, R_N\} \rightarrow 2^N$ or indirectly, that is, via the indirect approach $\{R_1, \ldots, R_N\} \rightarrow R \rightarrow 2^N$. It is only important that the individual preferences should somehow be aggregated so as to produce a set of options satisfying preferences of all involved parties according to some rationality principles. Here, we do not care if there are some intermediate steps in the process of choice. For example, the rule may first require creation of a group (collective) preference relation and only then—using this relation—select a set of options (the indirect approach). Moreover, some interesting and popular rules are meant just for producing group preference relations leaving the choice of a "best" options as irrelevant or obvious (e.g., social welfare functions—cf., Sen[41]). In cases where group preference relations are required to be linear orderings we will assume that the option(s) that is (are) first in that ordering is selected.

One of the most popular rules of aggregation is the simple majority rule (known also as the Condorcet rule)—confer Nurmi[8] for definitions of this and other choice rules dealt with later on. Basically, it is assumed to work for linear orderings and produce group linear ordering (what is not always possible, in general). Thus, this rule may be described by the following formulas:

$$R(s_i, s_j) \Leftrightarrow \text{Card}\ \{k : R_k(s_i, s_j)\} \geq \text{Card}\{k : R_k(s_j, s_i)\} \tag{34}$$

$$S_0 = \{s_i \in S : \underset{i \neq j}{\forall} R(s_i, s_j)\} \tag{35}$$

where Card$\{A\}$ denotes cardinality of the set A and S_0 is the set of collectively preferred options. As a counterpart for this rule in the fuzzy case Nurmi[44] proposed the following rule:

$$R(s_i, s_j)\,\text{card}\ \{\Leftrightarrow k : R_k(s_i, s_j) > \alpha\} \geq \textit{threshold} \tag{36}$$

$$\{s_i \in S : \neg \underset{j}{\exists} R(s_j, s_i)\} \tag{37}$$

Therefore, Nurmi[44] restated (34) adapting it to the case of a fuzzy relation R and employing a more flexible concept of majority (still crisp!) defined by a threshold. Note that in (37) still the strict quantifying is used (referred here to the concept of a nondomination).

Kacprzyk[14–16] interpreted rules (34) and (35) employing the concept of a fuzzy majority equated with a linguistic quantifier. He introduced the concept of a Q-core that may be informally stated in a slightly modified version, as the $Q1/Q2$-core (cf., Zadrożny[17]) as:

$CC_{Q1,Q2}$: *Set of options, which are for most (Q2) of individuals "better" than most (Q1) of the rest of options from the set S.*

$$CC_{Q1,Q2} \in F(S)$$

$$\mu_{CC_{Q1,Q2}}(s_i) = \underset{s_j}{Q1} \ \underset{e_k \in E}{Q2} \ R_k(s_i, s_j) \tag{38}$$

where F(S) denotes a family of all fuzzy sets defined on S. Then, using Zadeh's fuzzy linguistic quantifiers,[19] we obtain:

$$\begin{aligned} h_i^j &= \frac{1}{N}\sum_{k=1}^{N} r_{ij}^k \qquad h_i = \frac{1}{M-1}\sum_{\substack{j=1\\ j\neq i}}^{M} \mu_{Q_2}\big(h_j^i\big) \\ \mu_{CC_{Q1,Q2}}(s_i) &= \mu_{Q1}(h_i) \end{aligned} \tag{39}$$

where h_i^j denotes the degree to which, in the opinion of individuals, option s_i is better than option s_j; h_i denotes the degree to which, in the opinion of most (Q2) individuals, option s_i is better than all other options; $\mu_{Q1}(h_i)$ denotes the degree (to be determined) to which, in the opinion of most ($Q2$) individuals, option s_i is better than most ($Q1$) other options. Thus, the choice set's $CC_{Q1,Q2}$ membership function degree for an option s_i equals the truth value of the formula on the right-hand side of (38).

Formula (38) serves as a prototype for our generic CCR proposed in the next section. This aggregation scheme emerges from (38) as a result of three observations. First, varying the linguistic quantifiers employed we obtain different CCRs. Second, by changing the order of aggregation we can obtain another family of CCRs, among them again some other well-known rules. In what follows, some examples of well-known classical choice rules and their representation using our generic scheme are shown. Third, replacing the original Zadeh linguistic quantifiers with the OWA operators we obtain more flexible aggregation scheme. This leads us to the following transformation of (38) into:

$$\underset{s_j}{Q1} \ \underset{e_k \in E}{Q2} \ R_k(s_i, s_j) \rightarrow O_{most}^{j} O_{most}^{k} R_k(s_i, s_j)$$

In what follows j and k will be indexing the set of options and individuals, respectively.

Thus, $(O_{most}^{j} O_{most}^{k})$ denotes an OWA operator-guided aggregation over all options (individuals) and governed by the weight vector indicated by the lower index—here the *most.*

Now, the proposed generic CCR may be expressed as follows:

$$\mu_{CCR}(s_i) = O_1 O_2 R_k(s_p, s_q) \tag{40}$$

This scheme has a number of "degrees of freedom." Namely, specific CCRs may be recovered by deciding:

1. what are the upper indexes of the OWA operators, that is, if we first aggregate over individuals and then over options or in the reverse order,

2. what are the weights vectors of both OWA operators,
3. if the pair of options indexes (p, q) corresponds to (i, j) or to (j, i)

Thus, basically we can distinguish four types of CCRs:

Type I. $\mu_{CRR}(s_i) = O_1^k O_2^j R_k(s_i, s_j)$

Type II. $\mu_{CRR}(s_i) = O_1^j O_2^k R_k(s_i, s_j)$

Type III. $\mu_{CRR}(s_i) = O_1^k O_2^j R_k(s_j, s_i)$

Type IV. $\mu_{CRR}(s_i) = O_1^j O_2^k R_k(s_j, s_i)$

The rules of type III and IV should be properly understood: they identify the fuzzy sets of options that are collectively *rejected* by the group.

To check if our CCR really generalizes some classical rules (or concepts) in case of crisp preference relation, we have to apply appropriate defuzzification procedure, via the max or min operation. This may be done in the following way:

- for type I and II rules choose s_i such that $i = \arg\max_j \mu_{CCR}(s_j)$
- for type III and IV rules choose s_i such that $i = \arg\min_j \mu_{CCR}(s_j)$

Now we can point out some well-known rules covered by our generic scheme. In the sequel, we use some specific OWAs. Most of these rules originally assume the individual preferences as linear orderings and we will comment upon them in these terms.

First, there are rules, which may be classified as type I as well as type II:

1. $O_\forall O_\forall$ the "consensus solution" (41)

Thus, an option is a member of this collective choice set to a degree in which it dominates *all* options according to the preferences of *all* individuals. If, there is such an option with a relatively high membership degree, then it may be reasonably assumed as the consensual choice of the whole group. The following properties of this CCR may be easily observed:

- the resulting choice set may be non empty only for complete individual preference relations,
- in case the individuals' preferences are given as crisp linear orderings, this choice set is nonempty only if there is an option which is first in the orderings of all individuals.

2. $O_{avg} O_{avg}$ the Borda rule (42)

The CCR proposed by Borda for crisp orderings (called the *Borda count*) (see, e.g.,[51]). In this system an option gets M-1, M-2, M-3, ..., 0 points for being first, second, third, ..., last, respectively, in an individual ordering (M is the number of options under consideration, as previously). The points assigned to an option in particular orderings are then summed up, which yields the Borda count of the option. The alternative with the largest Borda count is declared the winner.

The CCR defined by (42) behaves for crisp orderings exactly like the Borda rule. For an option s_i the value of $O_{avg}\ (r_{i1}^k, r_{i2}^k, \ldots, r_{iM}^k) = (\sum_j r_{ij}^k)/M$ yields this options a "partial Borda count" (computed for the k-th individual) divided by M. Let us denote it as BC_i^k. Then $O_{avg}(BC_i^1, BC_i^2, \ldots, BC_i^N)$ yields option's s_i a "full" Borda count divided by $M * N$. Thus, the group ordering implied by the Borda rule is identical with the ordering implied by (42). In particular, an option is the winner according to the Borda rule if and only if it is the winner according to the rule (42).

Obviously it does not matter if we first aggregate over the options (as discussed above) or over the individuals as:

$$\sum_k \left(\sum_j r_{ij}^k \Big/ M \right) \Big/ N = \sum_j \left(\sum_k r_{ij}^k \Big/ N \right) \Big/ M = \sum_k \sum_j r_{ij}^k \Big/ (M * N)$$

The following rule may be classified as type III or IV:

3. $O_\exists O_\exists$- the minimax degree set (Nurmi)[44] (43)

The concept of the minimax degree set introduced by Nurmi is an extension of his minimax consensus winner defined (23). This set comprises such options that are *least dominated* when confronted with *all* other options in the preference relations of *all* individuals.

Some examples of type I rules are:

4. $O_{avg}^k\ O_\forall^j$- the plurality voting (44)

The classical plurality voting is defined for preferences expressed as crisp linear orderings of all options. An option is chosen if it appears as the first in the largest number of individual orderings. In (44) the OWA operator $O_\forall^j$ requires that an option dominates all other options and the operator O_{avg}^k "counts" for how many individuals it happens to be true. Thus, for crisp linear orderings $O_\forall^j(r_{i1}^k, \ldots, r_{iM}^k)$ returns 1 if option s_i is the first in the ordering of the individual k and 0 otherwise. Then the operator O_{avg}^k returns the number of "1" returned by $O_\forall^j(r_{i1}^k, \ldots, r_{iM}^k)$ for particular individuals k, divided by N—the total number of individuals.

5. $O_{maj}^k\, O_\forall^j$- the qualified plurality voting (45)

The qualified plurality voting extends the above-mentioned "regular" plurality voting in such a way that an option is chosen if and only if it is first in the orderings of a qualified majority of individuals. In (45) this additional requirement is properly modeled by the O^k_{maj} operator. The actual form of this operator depends, of course, on the specific form of required qualified majority.

$$6. \quad O^k_{avg} O^j_{maj}\text{- an approval voting-like rule} \tag{46}$$

In the approval voting procedure each individual chooses as many options as he or she likes. Then each option is ranked according to the number of individuals who have chosen it. The scheme defined by (46) is equivalent to the approval voting in the following sense. It is assumed that the individual preferences are crisp linear orderings and O^j_{maj} models the choice of an individual if he or she has to choose a subset of preferred options — as required by the classical approval voting procedure. For example, if we assume that each individual chooses first three options in her or his ordering then the O^j_{maj} operator has to correspond to the quantifier "at least M-3". Then the O^k_{avg} "counts" the rank of each option, as assumed in the classical approval voting procedure. Thus, the CCR (46) corresponds to the approval voting procedure under some specific assumptions as to the preferences and behavior of the individuals: the former is assumed to be represented by a linear ordering, whereas the latter boils down to picking up, by *all* individuals, the same number of first options in their orderings as those they choose.

$$7. \quad O^k_{\forall} O^j_{maj}\text{- the "consensus + approval voting" rule} \tag{47}$$

This is a CCR emerging from our generic scheme (40) that does not have a counterpart in the classical group decision-making literature. It may be characterized as a stronger version of the approval voting. Assuming the same voting procedure as for (46), it declares an option as a member of the collective choice only if it belongs to the sets of options picked up by *all* individuals.

Some examples of type II rules are:

$$8. \quad O^j_{\forall} O^k_{maj}\text{ - the simple majority (Condorcet)} \tag{48}$$

The classical rule of Condorcet (the majority rule) chooses such an option that dominates all other options in the majority of linear orderings expressed by particular individuals. The rule defined by (48) yields exactly the same result for crisp linear orderings.

$$9. \quad O^j_{\forall} O^k_{\exists}\text{ the Pareto optimal options} \tag{49}$$

An option s_i is *undominated in the Pareto sense* if there does not exist an option s_j such that $R^k(s_j,s_i)$ for all k and $P^k(s_j,s_i)$ for at least one k; P denotes here the strict preference, confer (4). In case of linear orderings of the individuals, the combination of OWA operators $O^j_{\forall} O^k_{\exists}$ requires exactly the same: for each option s_j there must be an individual k such that s_i precedes s_j in her or his ordering.

$$10. \quad O^j_{avg} O^k_{maj}\text{- the Copeland rule} \tag{50}$$

The Copeland choice set consists of those options that dominate the largest number of otherones in majority of linear orderings of the individuals. This is a weaker variation of the Condorcet choice rule giving higher chances for a resulting non empty choice set. In order to cover it with our generic scheme we replace the $O_\forall$ operator in (48) with the O_{avg} operator in (50). Here again O_{avg} makes it possible to "count" the number of options that are dominated by s_i in linear orderings of majority of individuals.

An example of a type III rule is:

11. $O^k_{most} O^j_{avg}$ Kacprzyk's Q-minimax set[14,16] (51)

Kacprzyk's Q-minimax set consists of these options which are dominated by smallest number of other options in the preferences of most of individuals. The formula (51) expresses exactly this idea.

And finally, some examples of type IV rules are

12. $O^j_\exists O^k_{avg}$ - the minimax set (Kramer[44]) (52)

Kramer's minimax set consists of those options which, when confronted with their toughest competitors, fare best, that is are dominated by them in smallest number of linear orderings of the individuals. To decide on the membership to this set for an option s_i, the OWA operator O^k_{avg} "counts" in how many orderings an option s_j precedes option s_i and the operator $O^j_\exists$ returns the highest of these numbers.

13. $O^j_\forall O^k_{maj}$ - the Condorcet looser (53)

The Condorcet looser is an option that is dominated by all other options in the majority of individual linear orderings. Such an option should not definitely be chosen. The formula (53) properly expresses this concept in case of linear orderings.

Thus, the generic scheme proposed covers some classical rules, especially well known in the context of voting. Some of the recovered rules are not CCRs *sensu stricto*. For example, rule 13 produces set of options that may be treated as collectively rejected rather than selected. For crisp preferences assumed in the context of these rules (usually linear orderings) our rules yield identical results. Moreover, these rules are readily applicable to any other forms of preference relations, notably fuzzy preference relations. The properties of rules obtained for these other forms of preference relations require further studies but we can expect they will reveal the aggregation behavior similar to that observed in the crisp context.

We can get a better insight into the *modus operandi* of the type II and IV rules using our previous analysis of the individual choice functions. These rules consist in, first, aggregation of individual preference relations into a *group preference relation* and, second, in choosing the subset of options on the basis of this relation. This is the *indirect* approach to collective choice, see, for example, Nurmi and Kacprzyk.[30] Herrera et al. (see, e.g., Herrera, Herrera-Viedma, and Verdegay[52]) recognize in this approach three steps: the *aggregation*, *exploitation*, and *selection*. Thus, the first OWA operator in a given CCR describes how group preference relation is formed and the second denotes how to choose, based on this group preference relation. The latter may be formally treated as the individual choice functions. The main difference consists in that we cannot guarantee that the properties assumed for individual preference relation are preserved by the aggregation scheme used in the first step. The particular rules for the formation of a group preference relation will be discussed as given below follows (for some more details, cf., Zadrożny and Kacprzyk[53]).

The individual choice rule max-MF (27) appears in the second step of the following CCRs: "consensus solution," the simple majority (Condorcet) and Pareto rules. The individual choice rule involved in max-SF (32) may be recognized in the Borda and Copeland CCRs. And finally, min-MA (28) appears in the minimax set (cf., Nurmi[44]). Thus, the counterparts of the individual choice functions may be treated as the building blocks of the CCRs that opens new lines of research.

6. CONCLUDING REMARKS

We have considered in this article an approach to a unified characterization of individual and collective choice functions, under fuzzy preferences and majority, that make it possible to find solutions best acceptable both by individuals and then by their group. This unified framework, obtained by using the concept of an OWA operator, is conceptually simple and can be used—by a simple adjustment process—to choose such a choice function that is proper in a particular case considered.

We may be able, therefore, to better manage knowledge and expertise of a group of experts and hence will be able to arrive at better decisions.

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