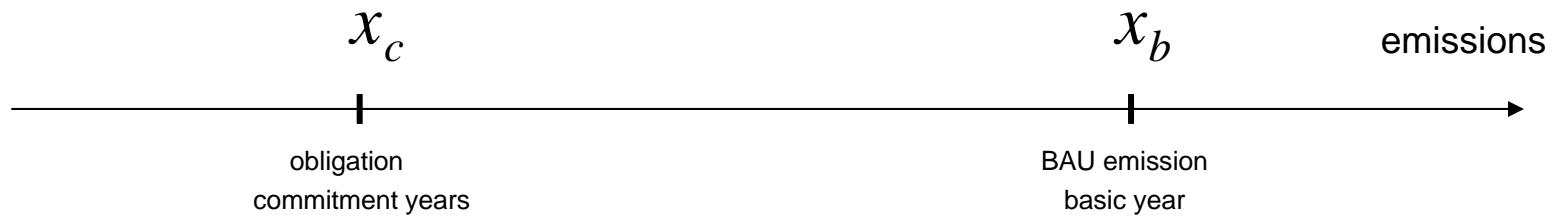


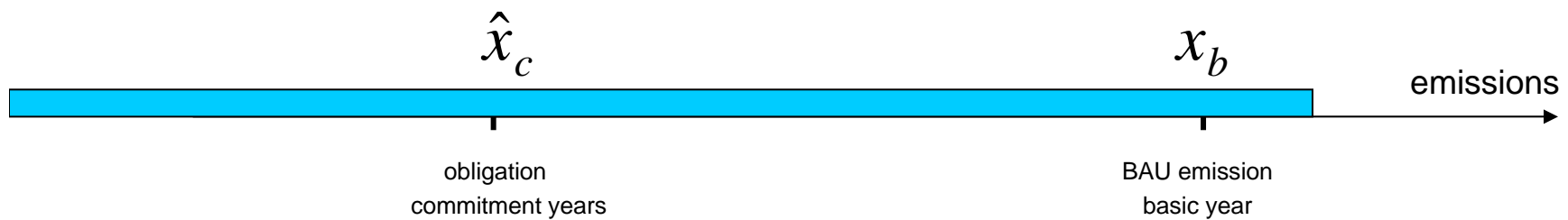
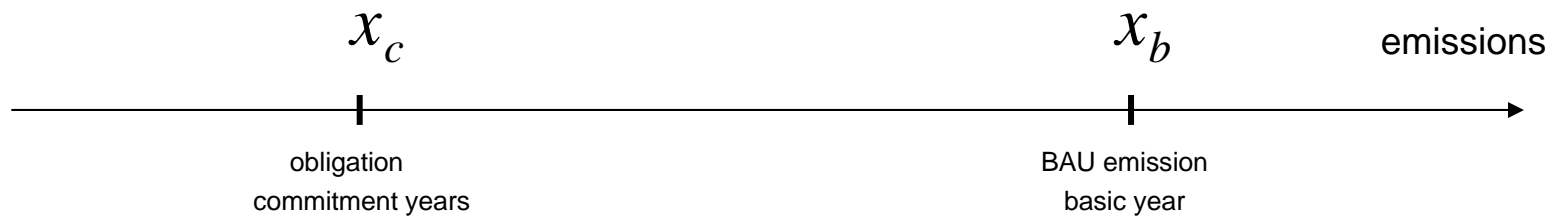
Compliance and Emission  
Trading Rules  
for Uncertain Estimates  
of Inventory Uncertainty

Zbigniew Nahorski & Joanna Horabik

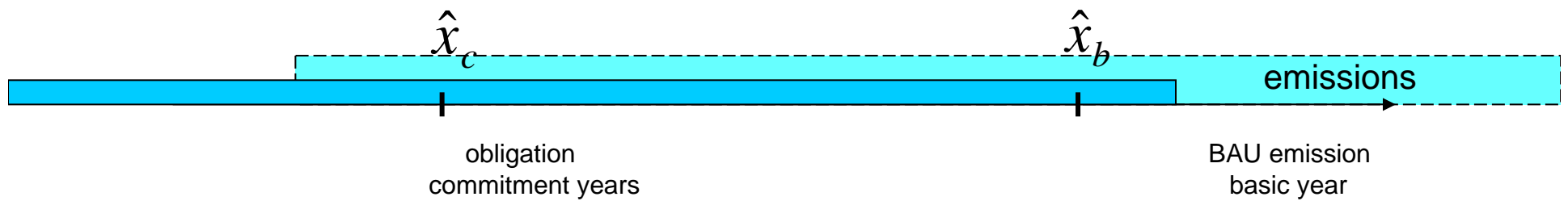
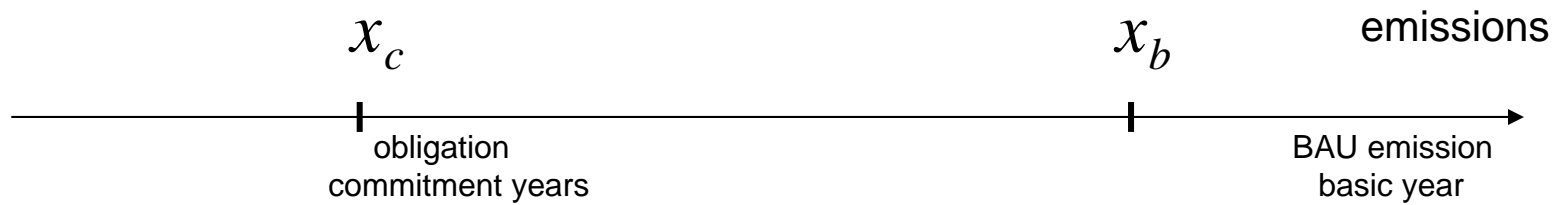
# Kyoto obligations



# Kyoto obligations



# Kyoto obligations



# Compliance under uncertainty (1)

$$x_c \leq (1 - \delta)x_b$$

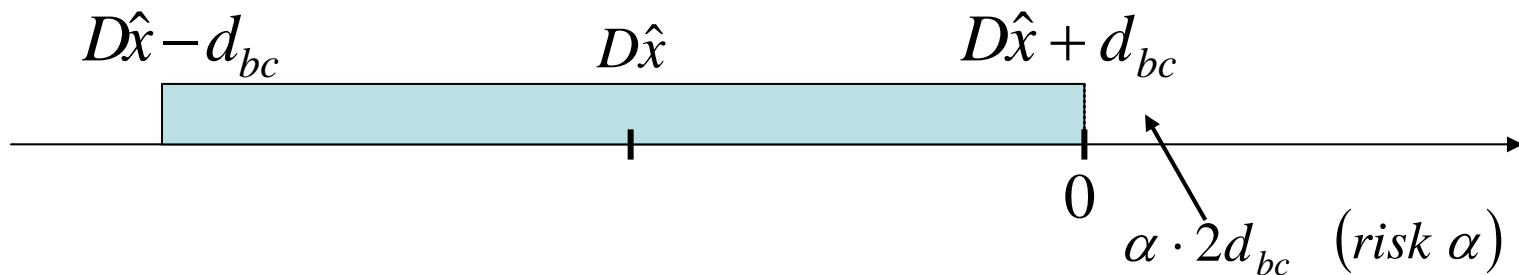
$$x_c - (1 - \delta)x_b \leq 0 \quad \text{exact knowledge}$$

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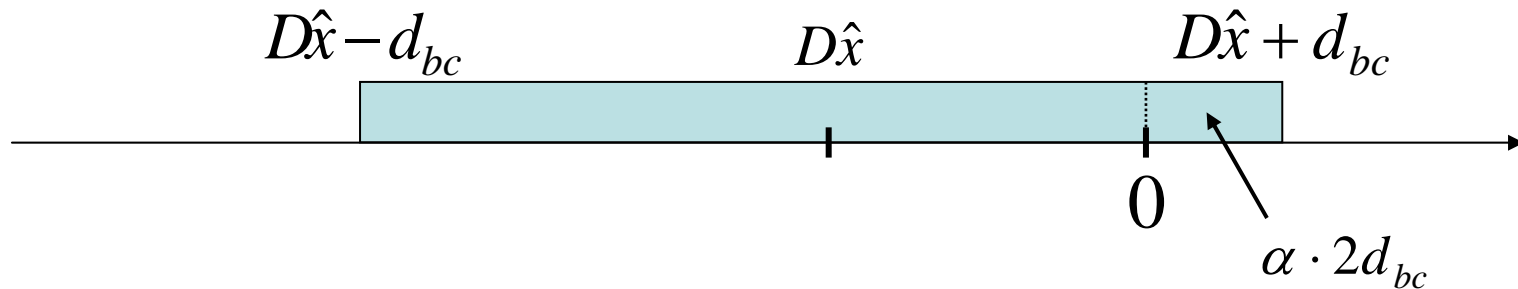
$$x_c \in [\hat{x}_c - d_c, \hat{x}_c + d_c] \quad x_b \in [\hat{x}_b - d_b, \hat{x}_b + d_b]$$

$$x_c - (1 - \delta)x_b \in [D\hat{x} - d_{bc}, D\hat{x} + d_{bc}]$$

$$D\hat{x} = \hat{x}_c - (1 - \delta)\hat{x}_b$$
$$d_{bc} = d_c + (1 - \delta)d_b$$



# Compliance under uncertainty (2)



$$\hat{x}_c - (1 - \delta)\hat{x}_b + d_{bc} \leq 2\alpha d_{bc}$$

$$\boxed{\hat{x}_c} + \boxed{(1 - 2\alpha)d_{bc}} \leq (1 - \delta)\hat{x}_b$$

inventory

unreported inventory  
due to uncertainty

modified compliance rule

# Emission trading

Buying party before the trade

$$\hat{x}_c^B + (1 - 2\alpha)d_{bc}^B \leq (1 - \delta)x_b^B$$

purchased quota  $\hat{E}^S$  with uncertainty  $\hat{E}^S R_c^S$

Selling party relative uncertainty

$$R_c^S = \frac{d_c^S}{\hat{x}_c^S}$$

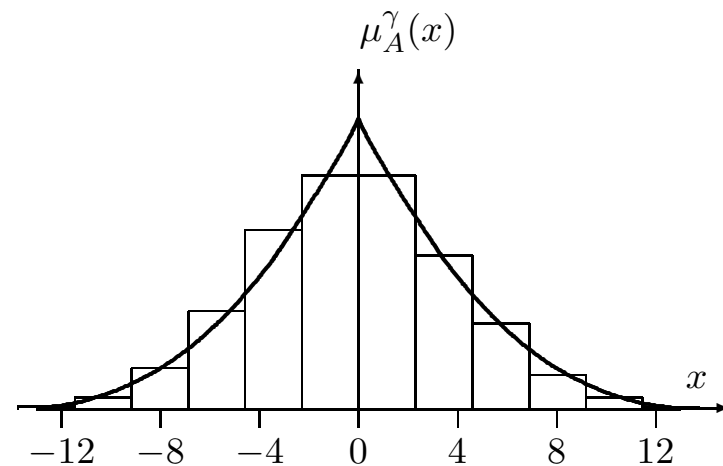
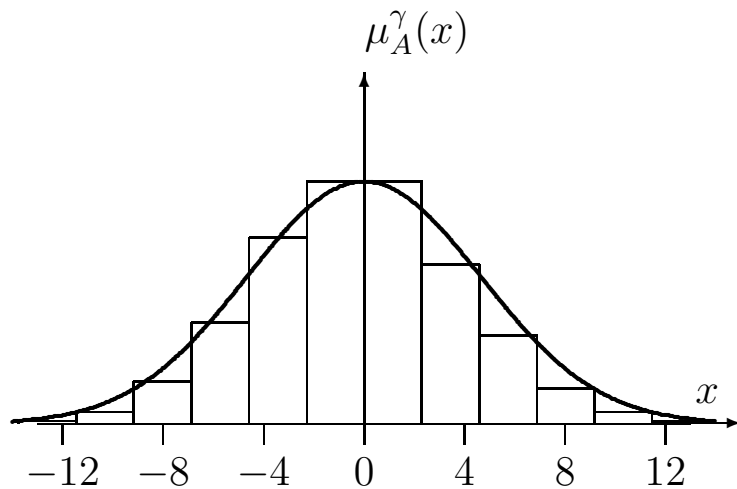
Buying party after the trade

$$\hat{x}_c^B - \hat{E}^S + (1 - 2\alpha)\left[d_{bc}^B + \hat{E}^S R_c^S\right] \leq (1 - \delta)x_b^B$$

real difference

$$\left[1 - (1 - 2\alpha)R_c^S\right]\hat{E}^S = E_{eff} \quad \text{effective emission}$$

# Inventory uncertainty distribution

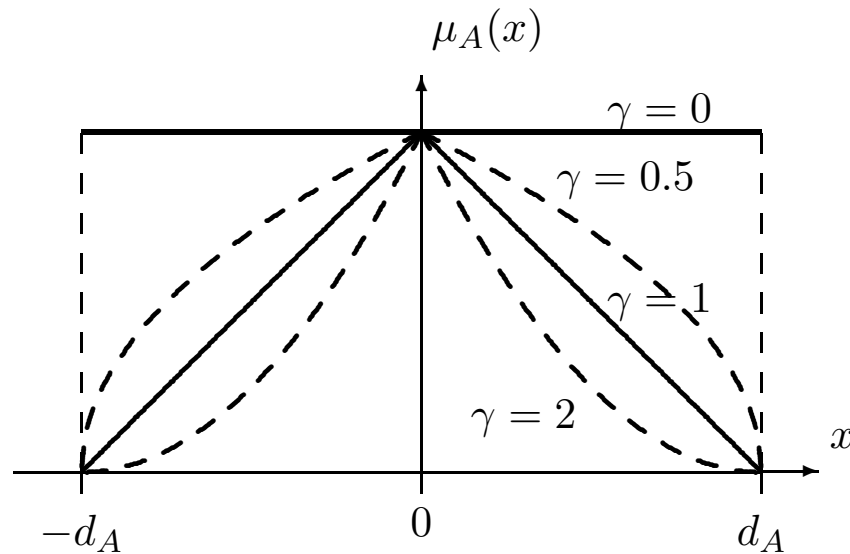


membership function  $\longrightarrow \mu^\gamma(x) = C \left( 1 - \frac{|x - \hat{x}|}{d} \right)^\gamma$

$$\gamma = 2.43$$



# A fuzzy number



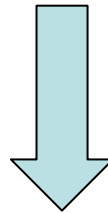
a fuzzy number

$$\{x, \mu^\gamma(x)\}$$

Need not be symmetric

# Compliance with the fuzzy numbers

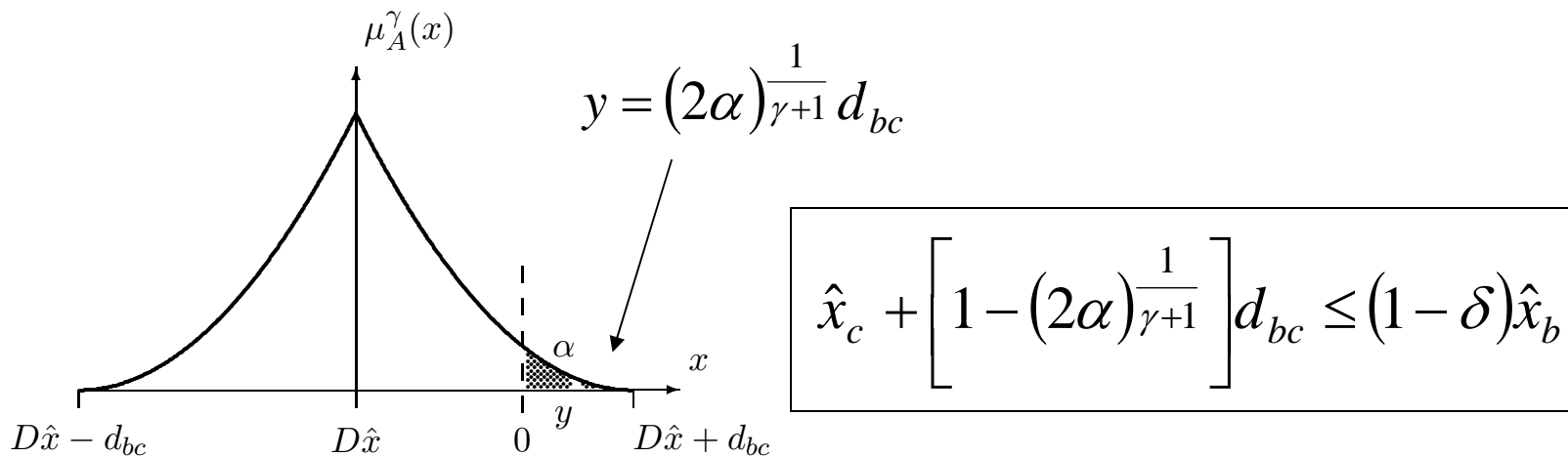
$$\left\{ \hat{x}_b, \left( 1 - \frac{|x - \hat{x}_b|}{d_b} \right)^\gamma \right\} \quad \left\{ \hat{x}_c, \left( 1 - \frac{|x - \hat{x}_c|}{d_c} \right)^\gamma \right\}$$



algebra like in the interval case

$$\left\{ D\hat{x}, \left( 1 - \frac{|x - D\hat{x}|}{d_{bc}} \right)^\gamma \right\}$$

# Compliance rule



# Emission trading

$$E_{eff} = \left\{ 1 - \left[ 1 - (2\alpha)^{\frac{1}{\gamma+1}} \right] \right\} \hat{E}^S$$

$$E_{eff} / \hat{E}^S \text{ for } \alpha = 0.3$$

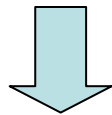
country	$\delta$	inter.	fuzzy				
	[%]	$\gamma=0$	$\gamma=0.5$	$\gamma=1$	$\gamma=1.5$	$\gamma=2$	$\gamma=2.5$
Austria	12	0.904	0.965	0.973	0.978	0.981	0.984
Holland	5	0.960	0.986	0.989	0.991	0.992	0.993
Norway	21	0.832	0.939	0.953	0.961	0.967	0.971
Poland	6	0.952	0.983	0.986	0.989	0.991	0.992
Russia	17	0.864	0.951	0.962	0.969	0.973	0.977
U.K.	19	0.848	0.945	0.957	0.965	0.970	0.974

# Equivalence of interval and fuzzy approach (1)

Equivalence:

- the same reduction,
- the same effective emission

$$(2\alpha_I)^{1+\gamma} = 2\alpha_F$$



$$\alpha_I \geq \alpha_F$$

# Equivalence of interval and fuzzy approach (2)

		$\gamma$		
		1.5	2	2.5
$\alpha_F$	$\alpha_I$			
	0.05	0.20	0.23	0.26
	0.10	0.26	0.29	0.32
	0.15	0.31	0.33	0.35
	0.20	0.35	0.37	0.38

# Conclusion

Use interval rules  
with  $\alpha = 0.25 - 0.30$

Thank you for attention